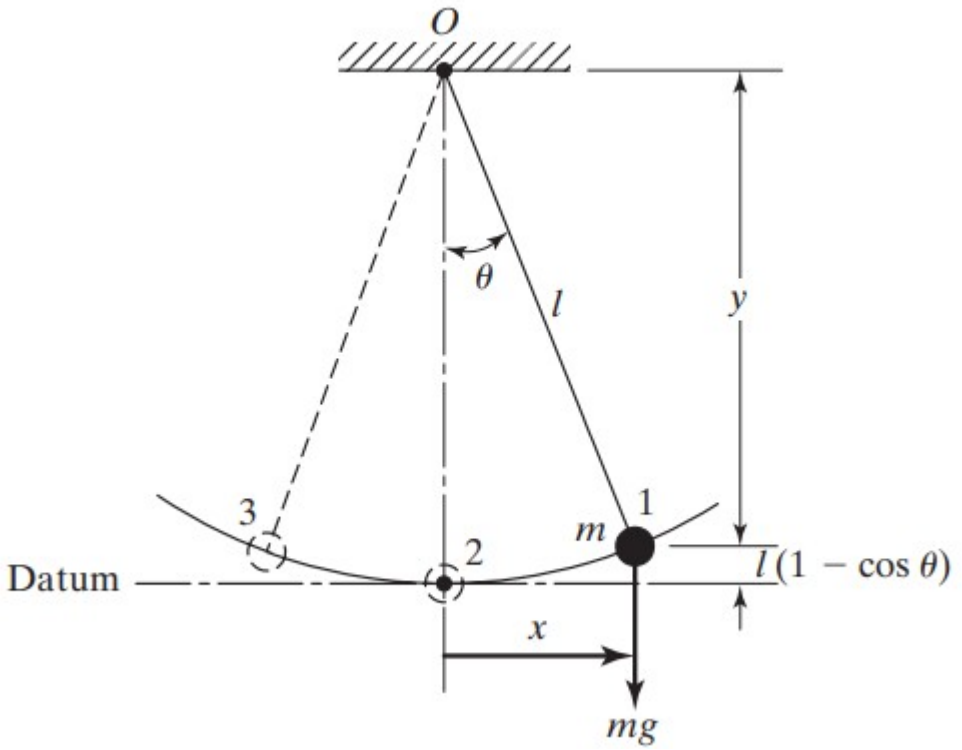
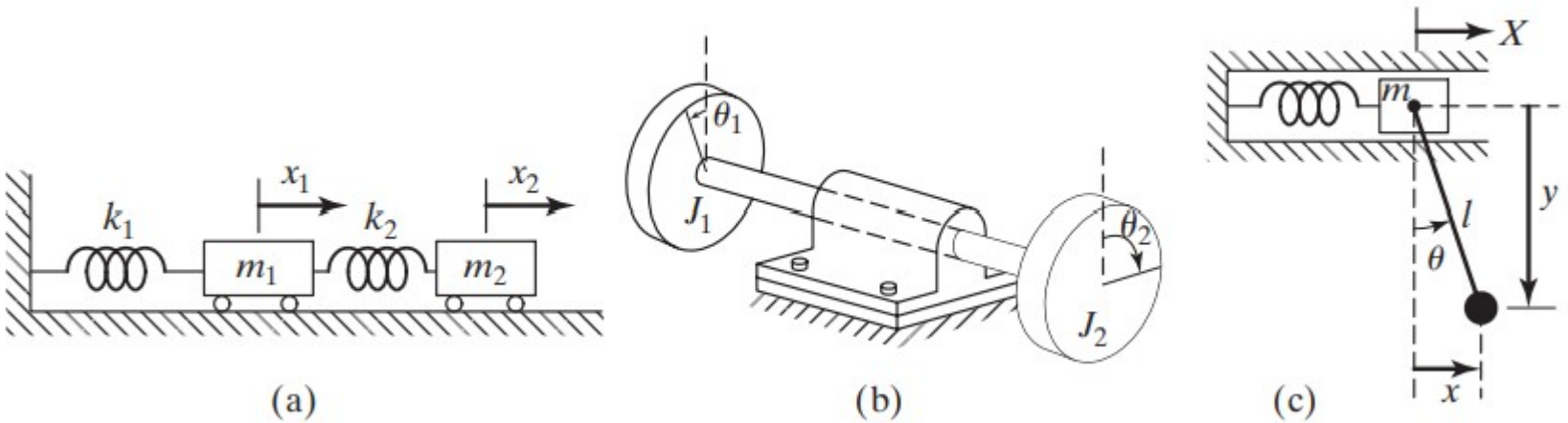
Drgania w konstrukcjach lotniczych

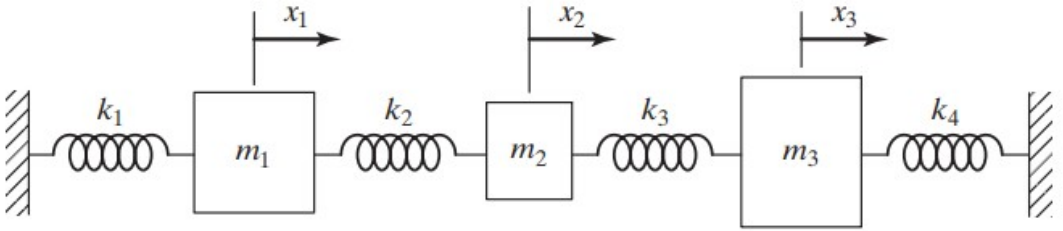
Proste wahadło



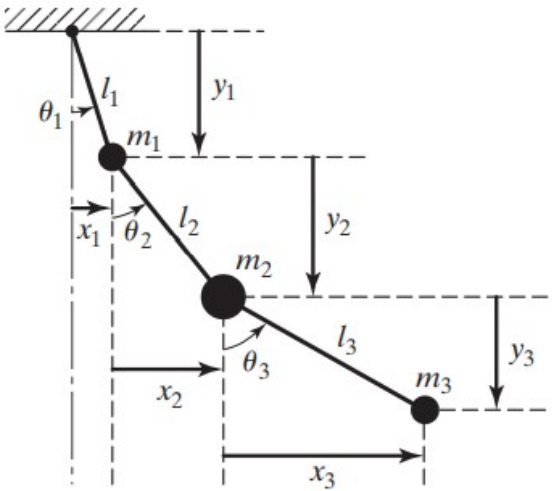
Proste wahadło



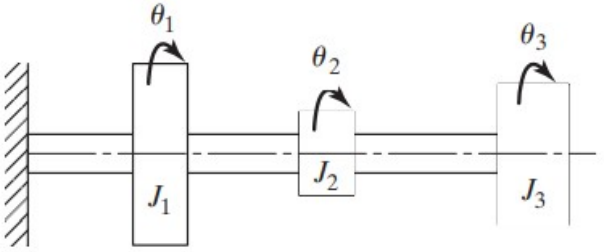
Układ o wielu stopniach swobody



(a)

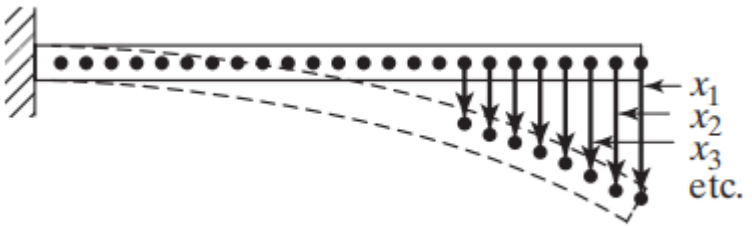


(b)

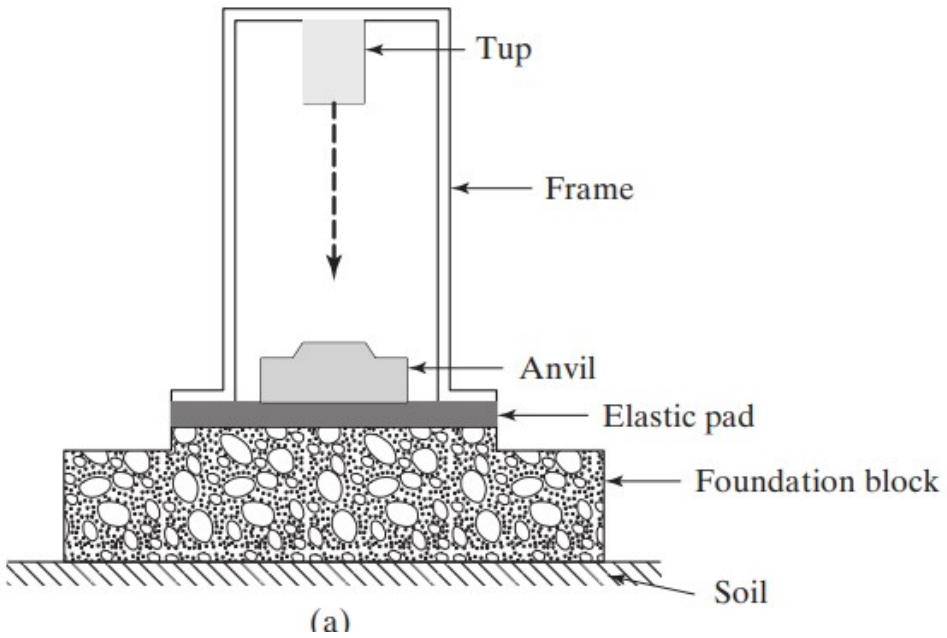


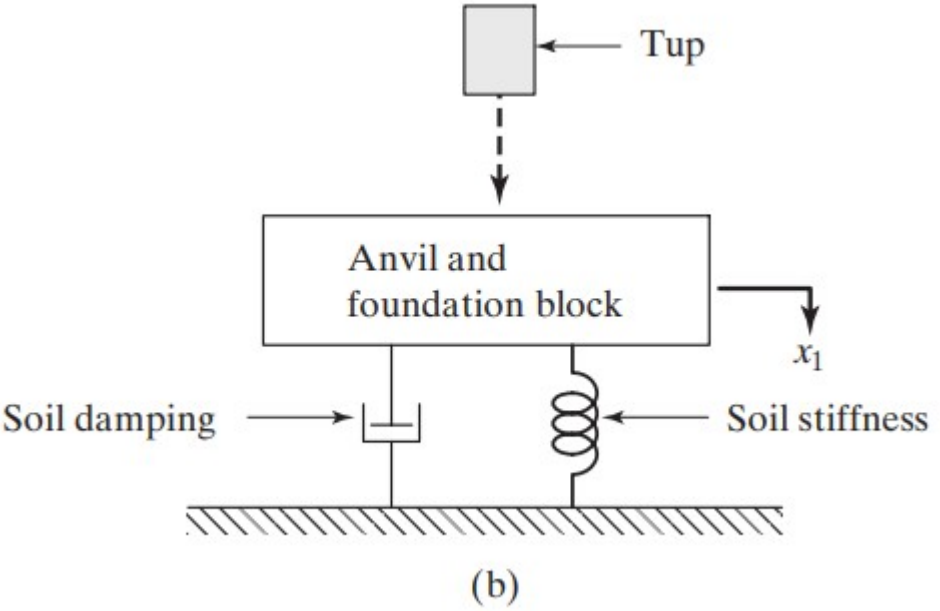
(c)

Układ o nieskończenie wielu stopniach swobody

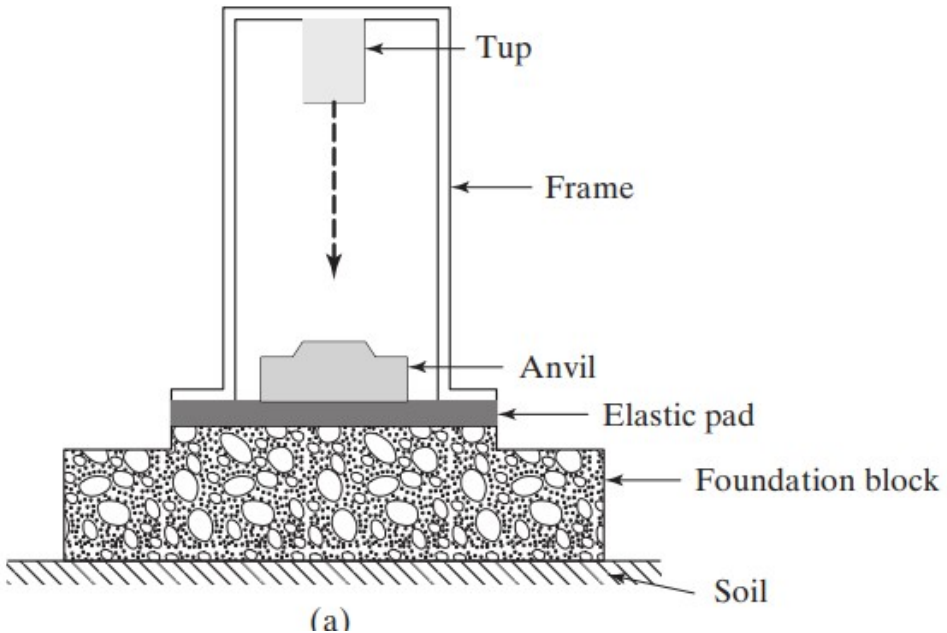


Redukcja układu fizycznego do modelu

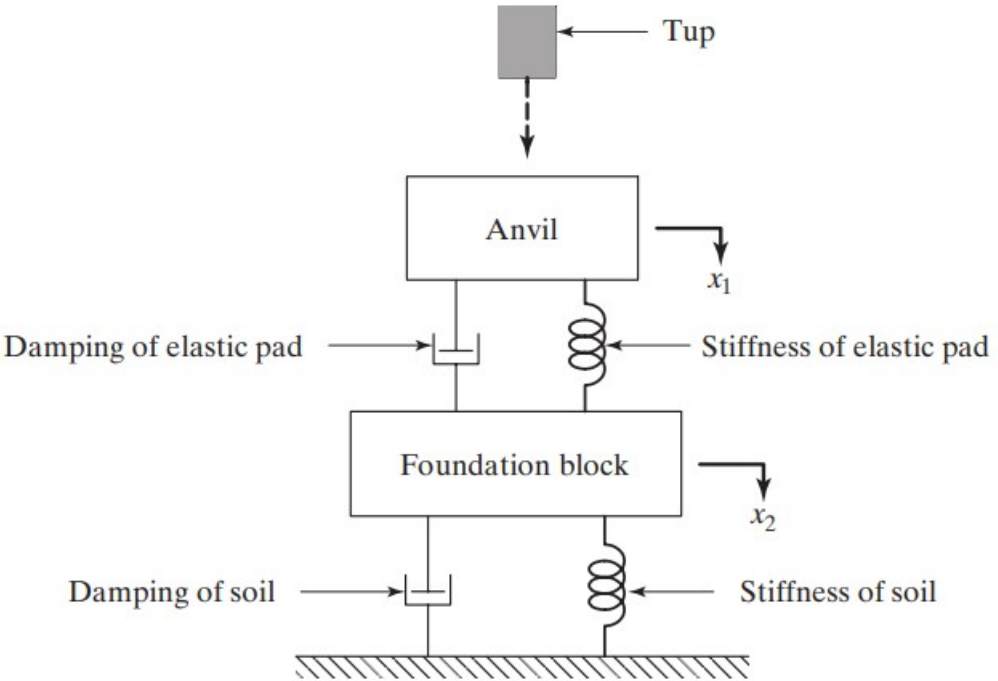




Redukcja układu fizycznego do modelu

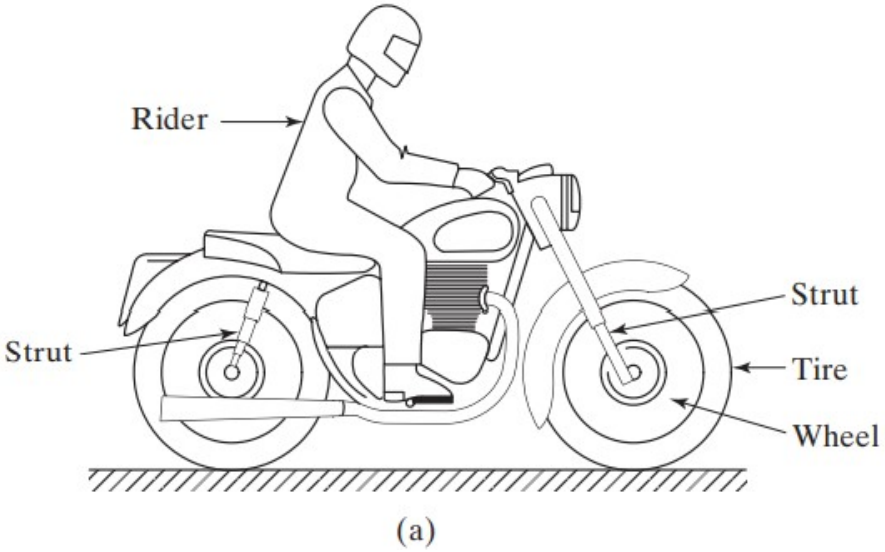


(a)

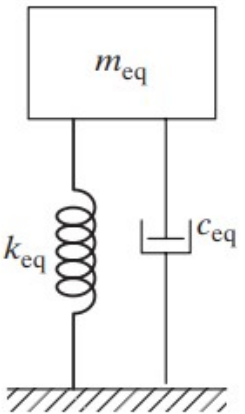


(c)

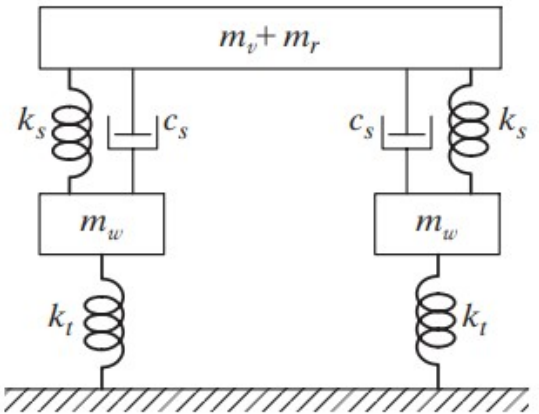
Redukcja układu fizycznego do modelu



(a)

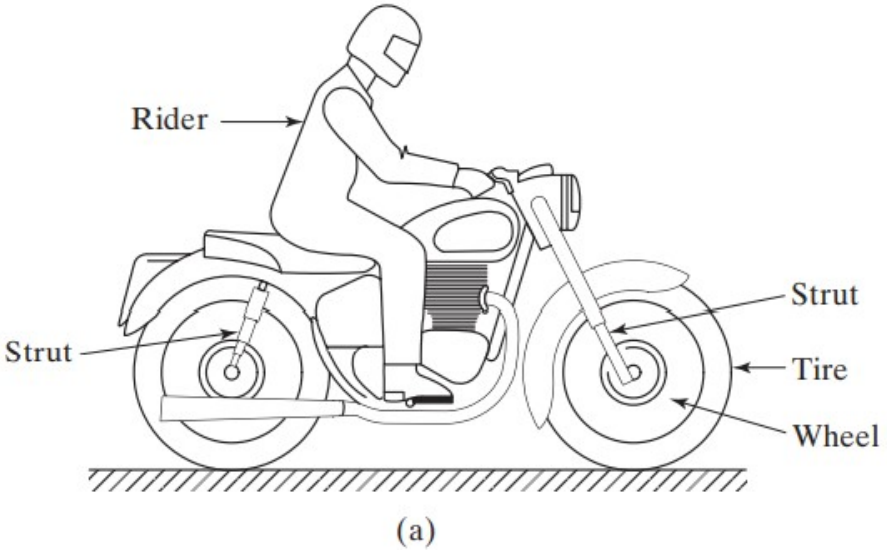


(b)

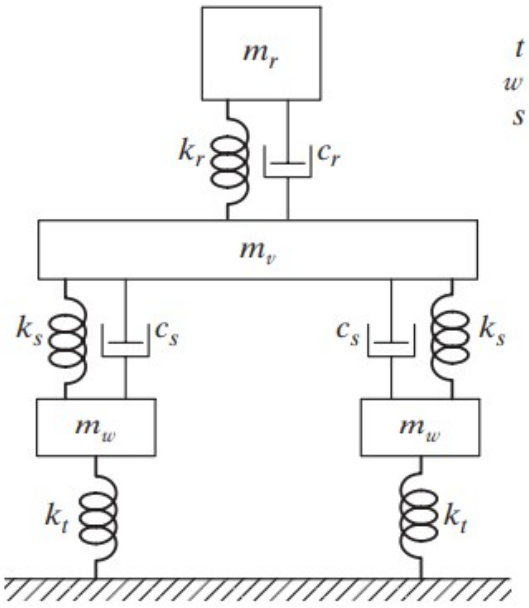


(c)

Redukcja układu fizycznego do modelu

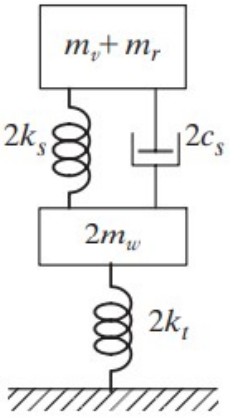


(a)



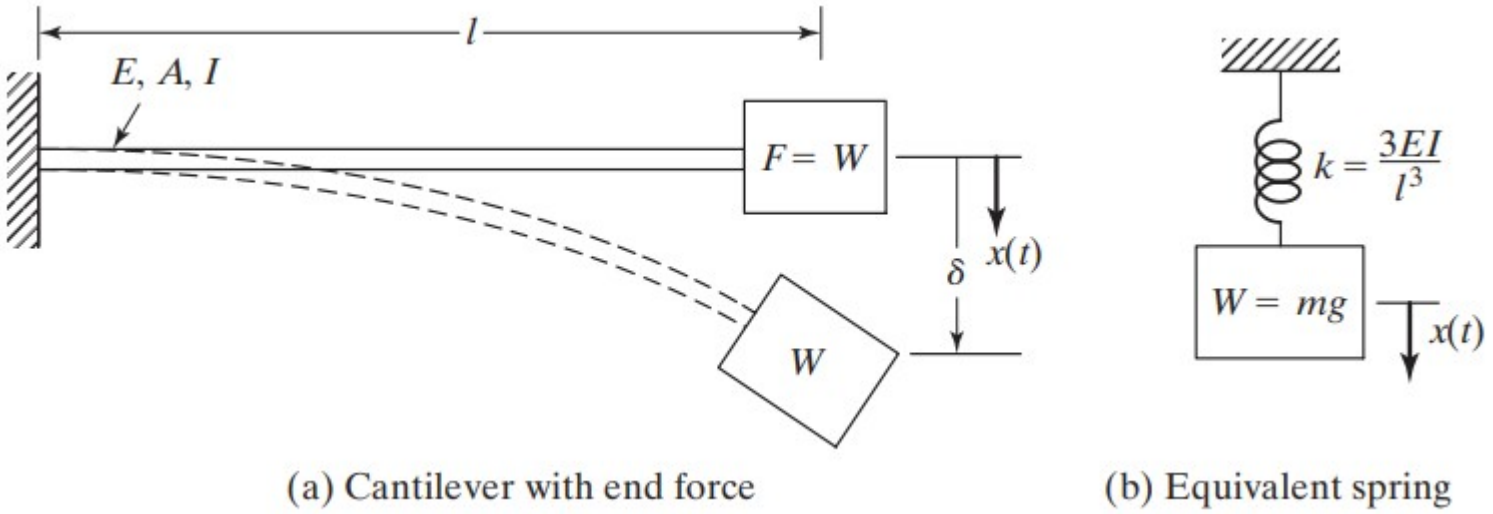
(d)

Subscripts
t : tire v : vehicle
w : wheel r : rider
s : strut eq : equivalent



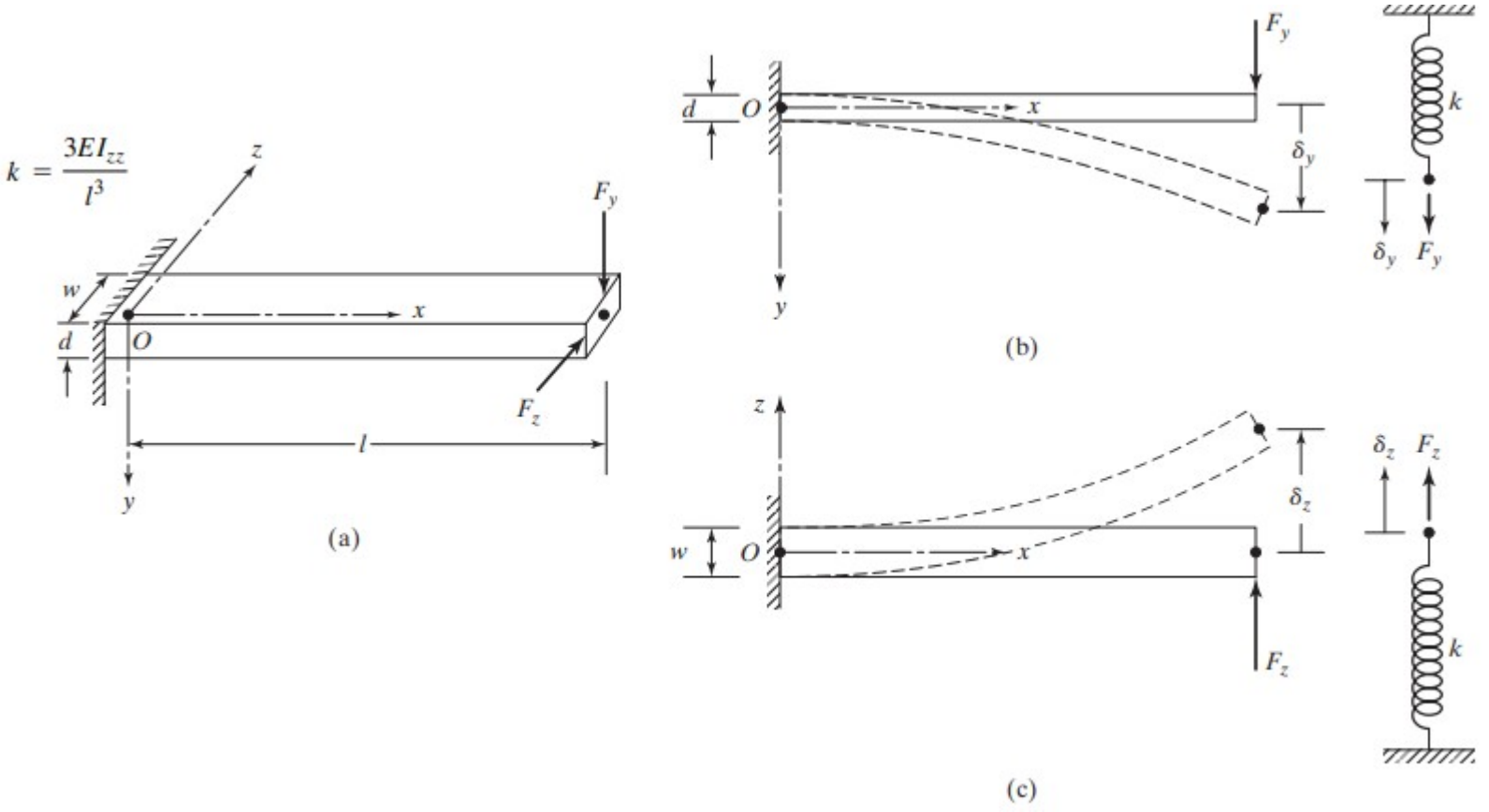
(e)

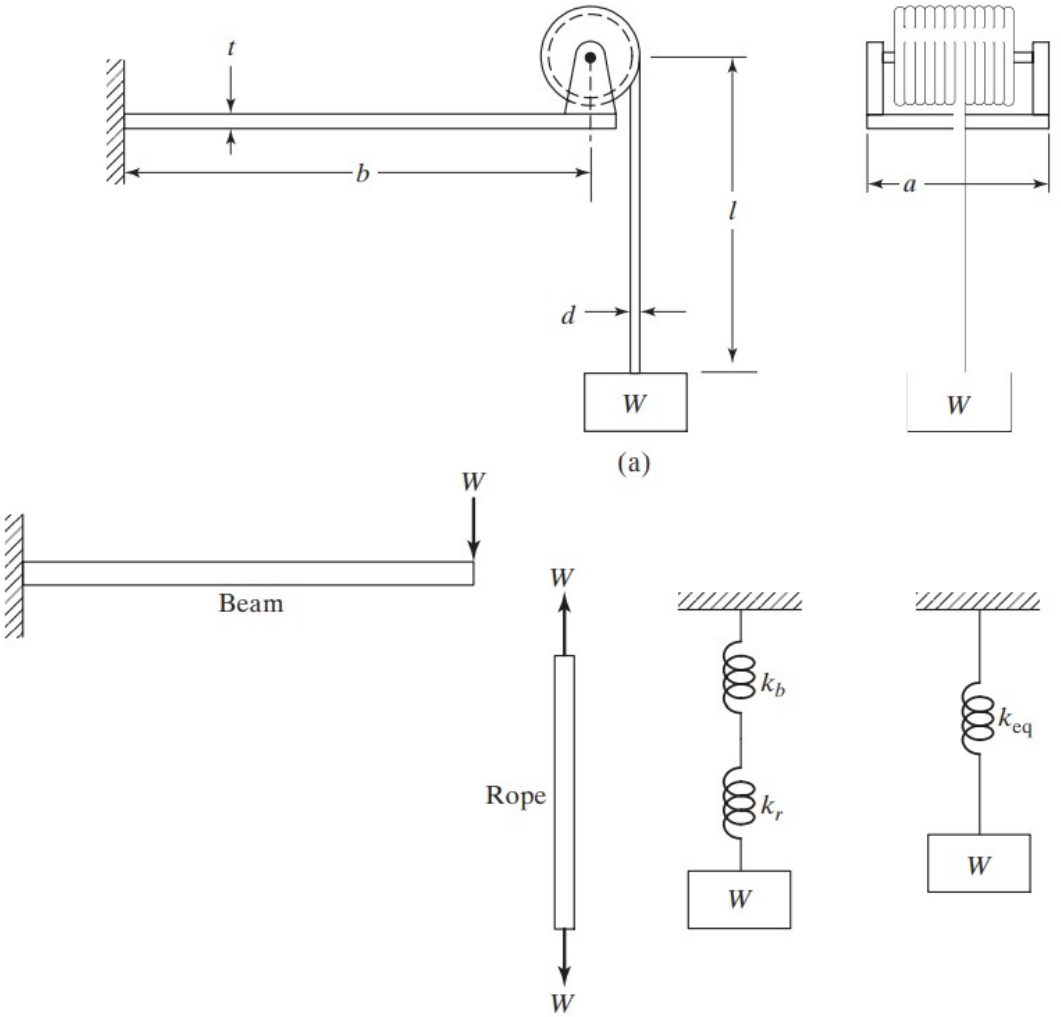
Równoważność różnych układów fizycznych

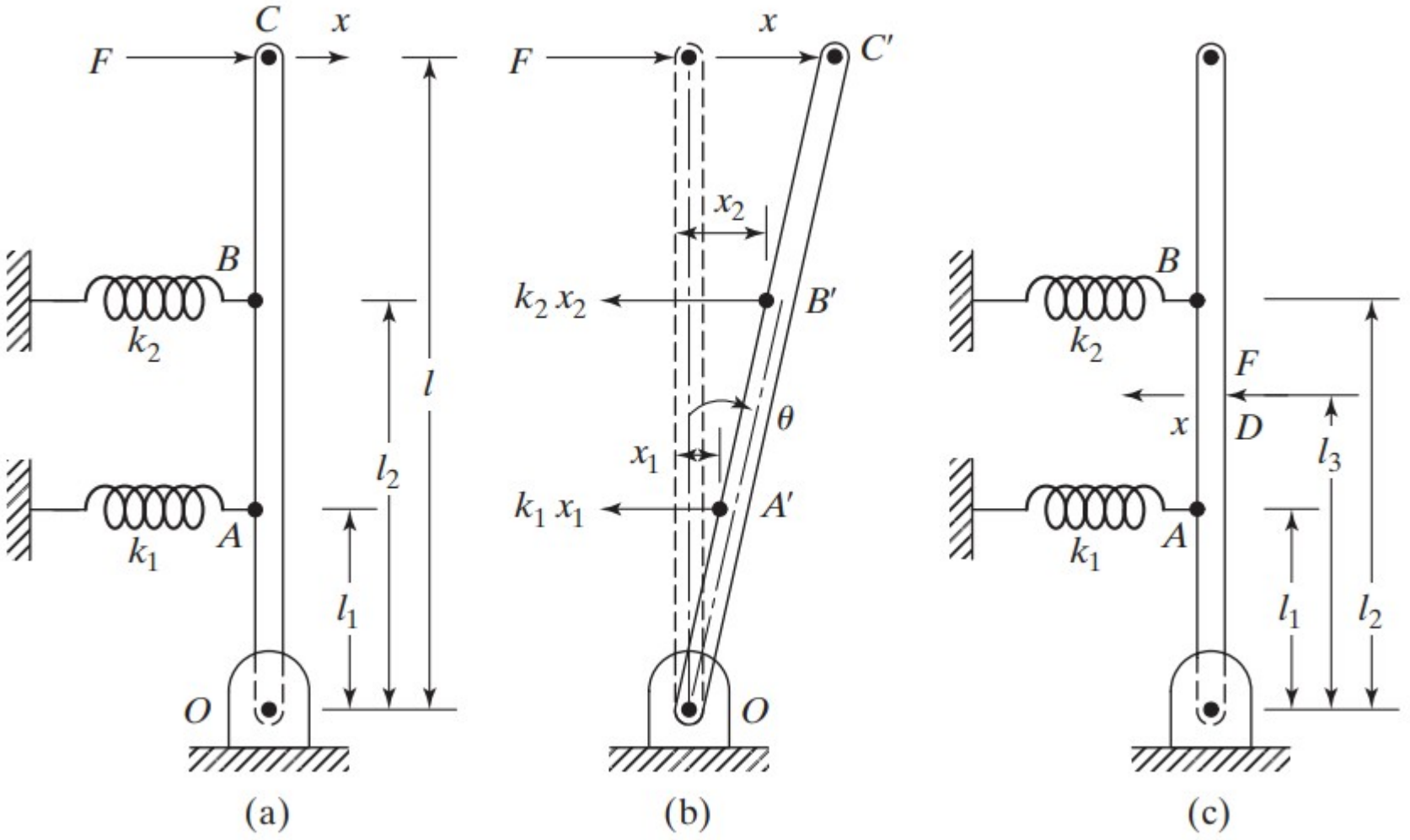


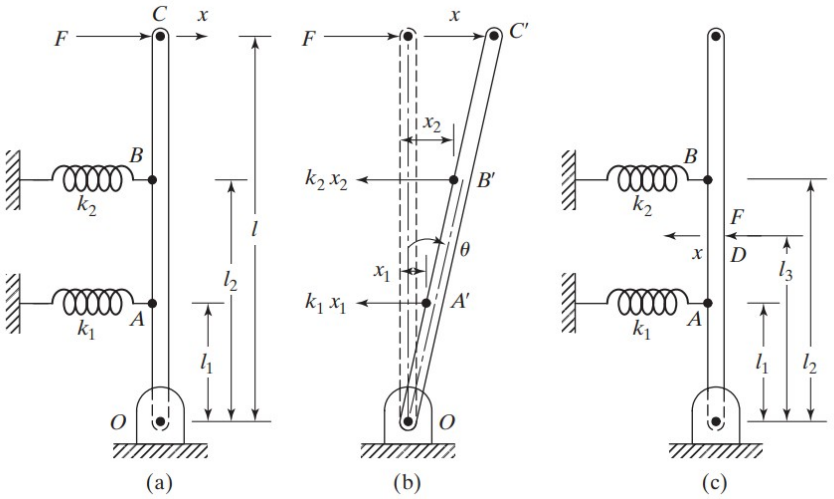
(a) Cantilever with end force

(b) Equivalent spring


$k = \frac{3EI_{zz}}{l^3}$
z
F_y
w
x
d
O
F_z
l
y
(a)
F_y
d
O
x
δ_y
k
δ_y
F_y
y
(b)
z
δ_z
F_z
w
O
x
δ_z
F_z
k
(c)


t
b
l
d
W
a
W
(a)
W
Beam
W
Rope
W
(b)
k_b
k_r
W
(c)
k_{eq}
W
(d)


C
x
F
B
k_2
l
l_2
k_1
A
l_1
O
(a)
x
F
C′
x_2
$k_2 x_2$
B′
θ
x_1
$k_1 x_1$
A′
O
(b)
B
k_2
F
x
D
l_3
k_1
A
l_1
l_2
(c)



$$k_1 x_1(l_1) + k_2 x_2(l_2) = F(l)$$

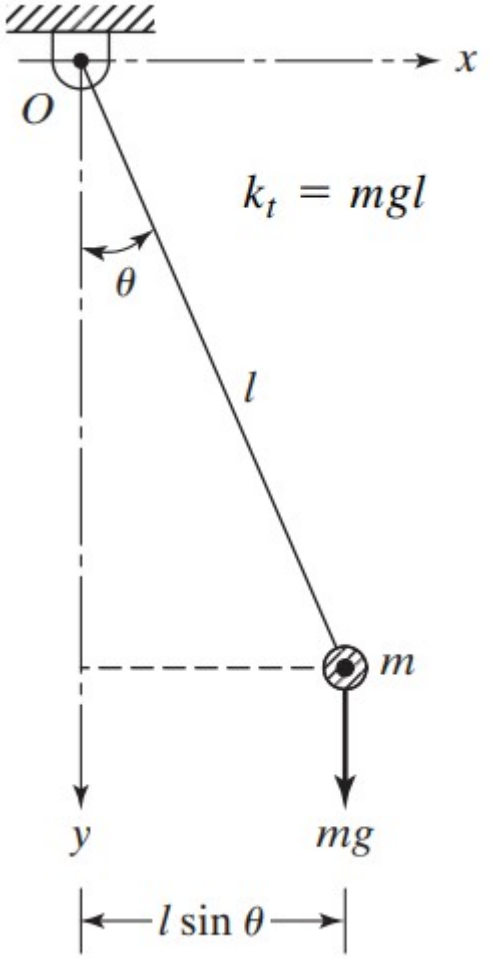
$$F = k_1\left(\frac{x_1 l_1}{l}\right) + k_2\left(\frac{x_2 l_2}{l}\right)$$

$$F = k_{eq}x = k_1\left(\frac{x_1 l_1}{l}\right) + k_2\left(\frac{x_2 l_2}{l}\right)$$

$$x_1 = l_1\theta,\ x_2 = l_2\theta, \text{ and } x = l\theta,$$

$$k_{eq} = k_1\left(\frac{l_1}{l}\right)^2 + k_2\left(\frac{l_2}{l}\right)^2$$

Zastępcza stała sprężystości

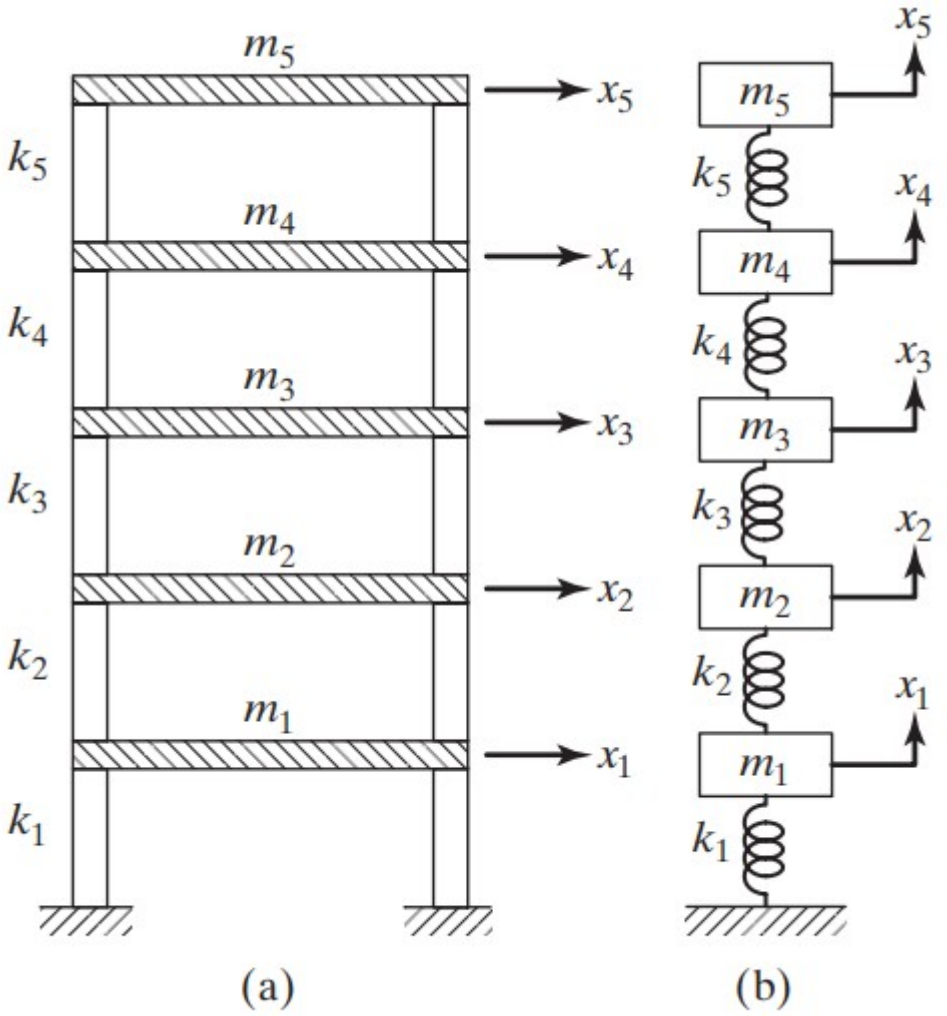


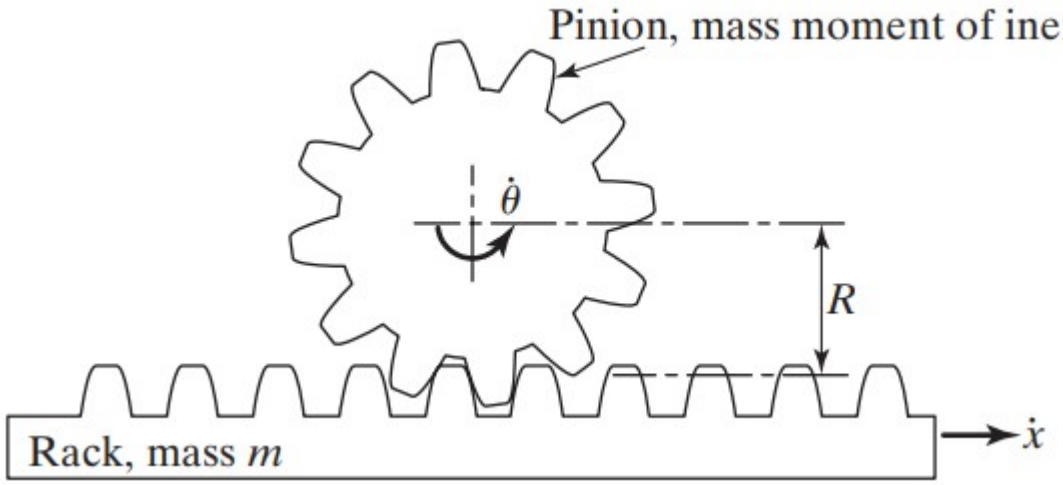
$$T = mg(l \sin \theta)$$

$$\mathrm{T} = mgl\theta$$

$$T = k_t\theta$$

$$k_t = mgl$$


m_5
x_5
k_5
m_4
x_4
k_4
m_3
x_3
k_3
m_2
x_2
k_2
m_1
x_1
k_1
(a)
x_5
m_5
k_5
x_4
m_4
k_4
x_3
m_3
k_3
x_2
m_2
k_2
x_1
m_1
k_1
(b)

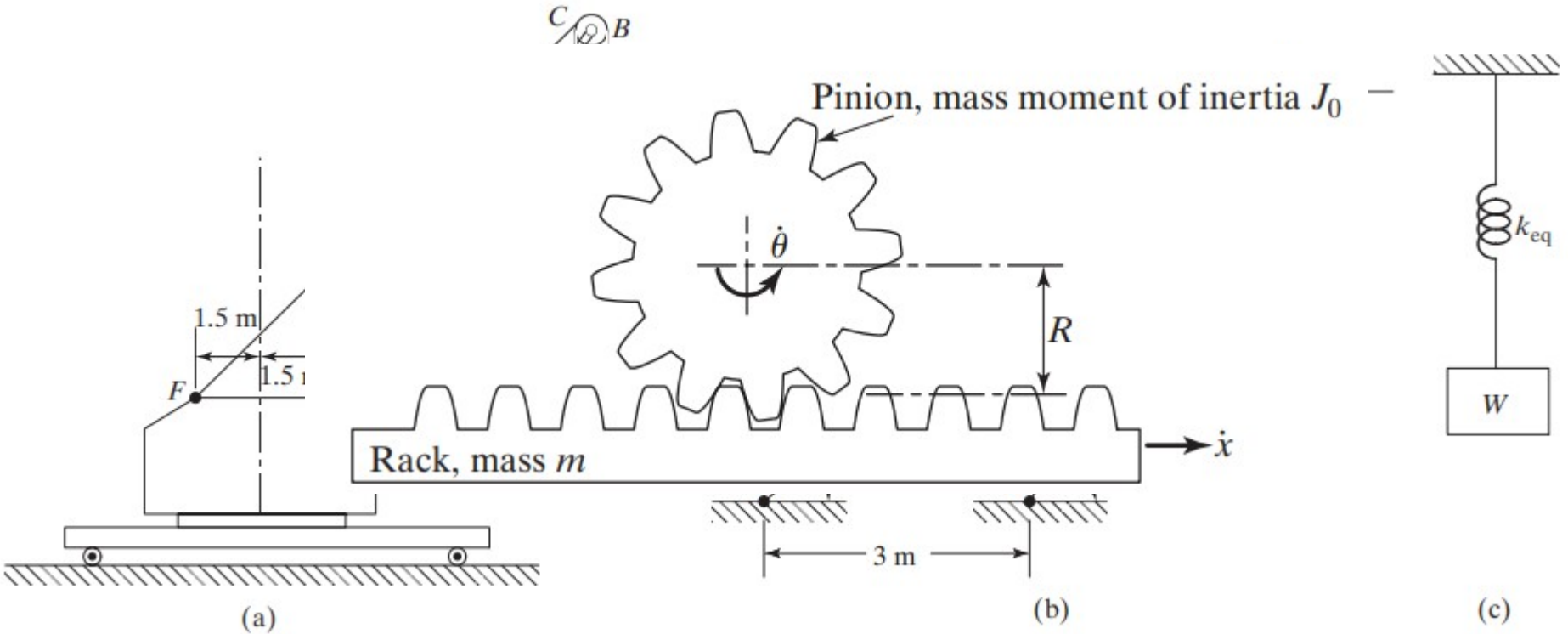


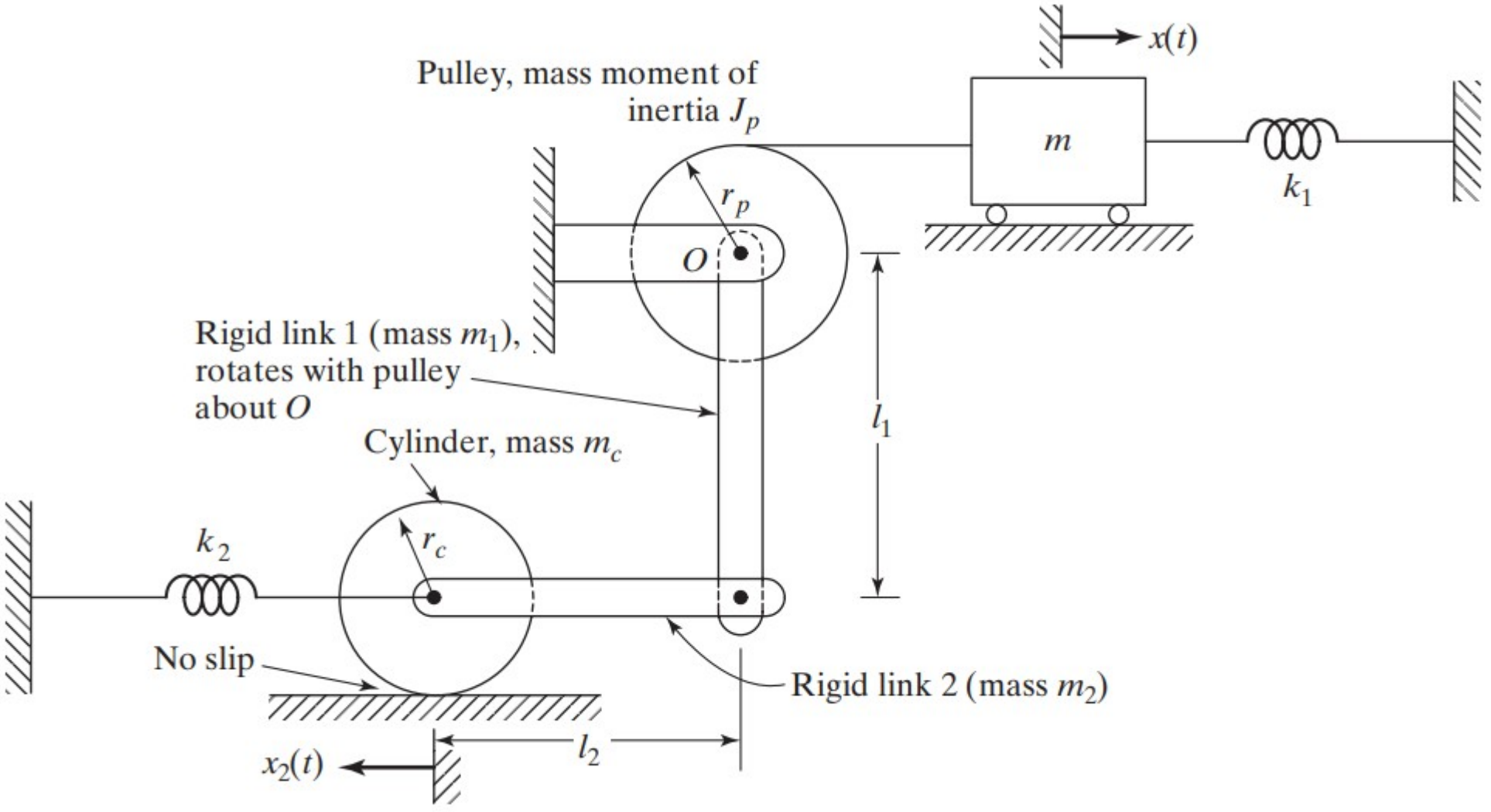
$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2$$

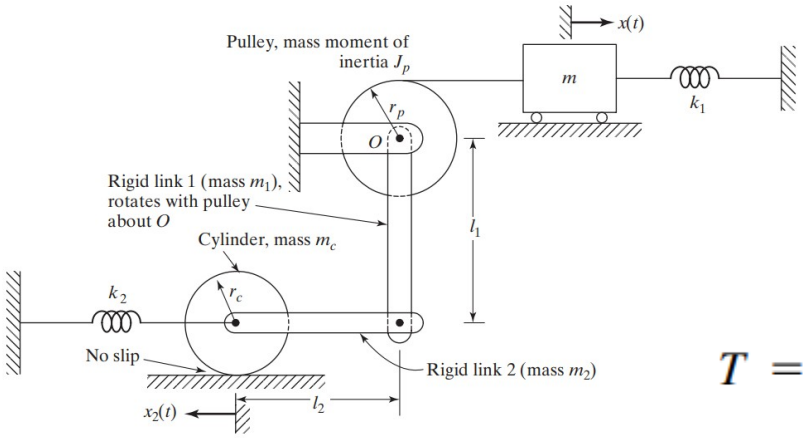
$$T_{eq} = \frac{1}{2}m_{eq}\dot{x}_{eq}^2$$

$$\dot{x}_{eq} = \dot{x} \quad \dot{\theta} = \dot{x}/R,$$

$$\frac{1}{2}m_{eq}\dot{x}^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\left(\frac{\dot{x}}{R}\right)^2$$


C
B
Pinion, mass moment of inertia J_0
$\dot{\theta}$
R
1.5 m
1.5
F
Rack, mass m
$\dot{x}$
3 m
k_{eq}
W
(a)
(b)
(c)


x(t)
Pulley, mass moment of inertia J_p
m
k_1
r_p
O
Rigid link 1 (mass m_1), rotates with pulley about O
l_1
Cylinder, mass m_c
k_2
r_c
No slip
Rigid link 2 (mass m_2)
l_2
$x_2(t)$



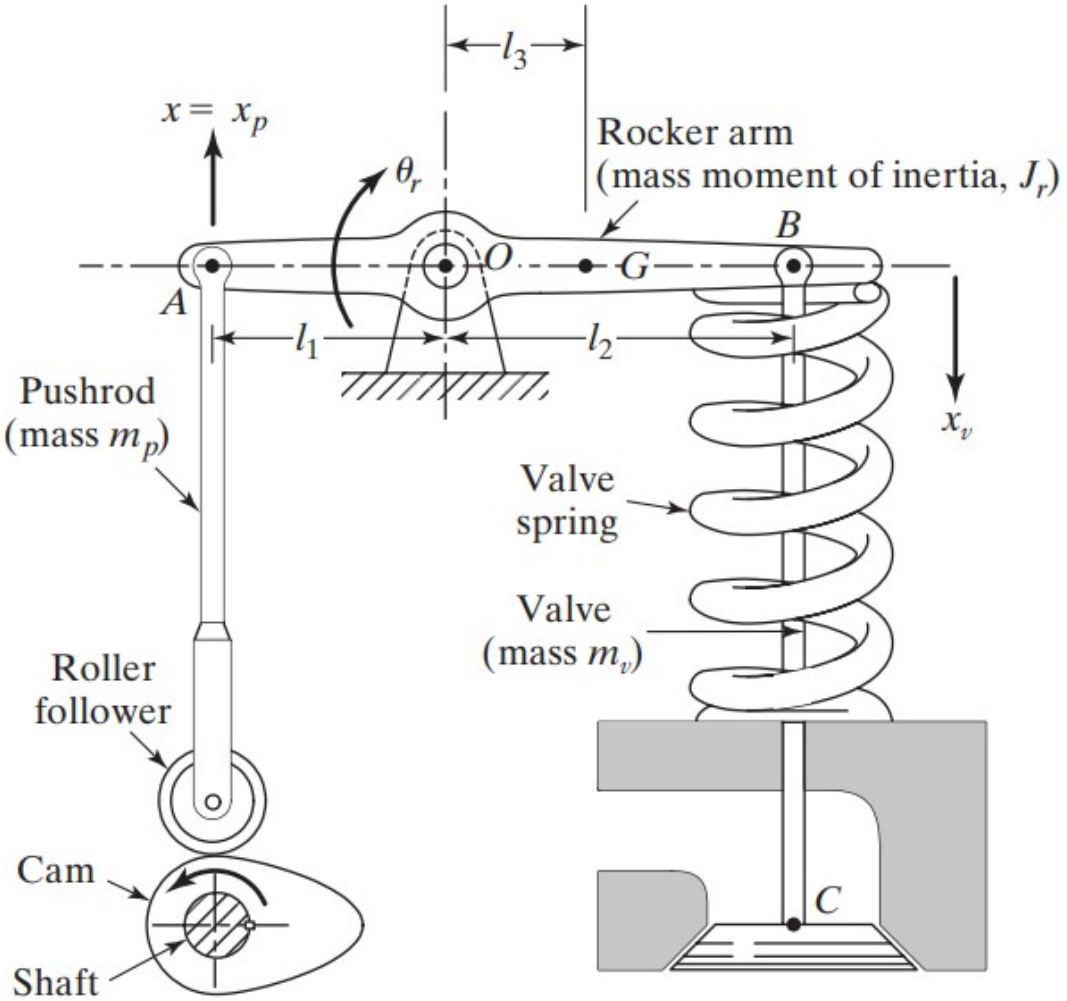
$$J_c = m_c r_c^2/2 \qquad J_1 = m_1 l_1^2/3,$$

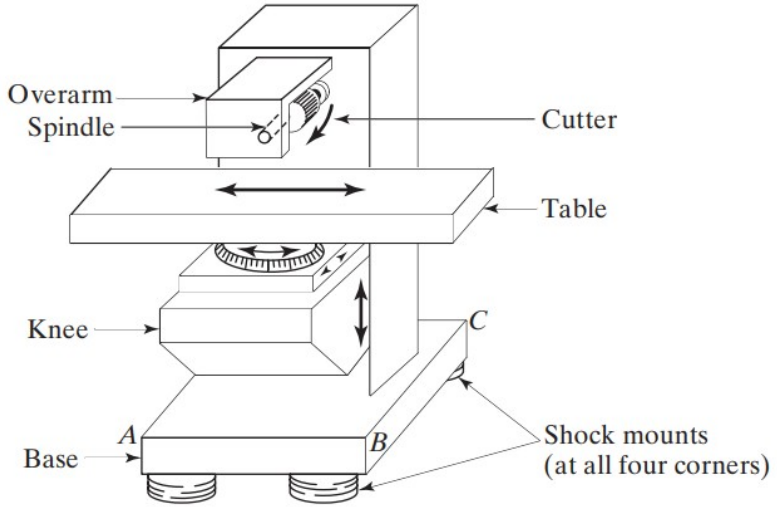
$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_p\dot{\theta}_p^2 + \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}J_c\dot{\theta}_c^2 + \frac{1}{2}m_c\dot{x}_2^2$$

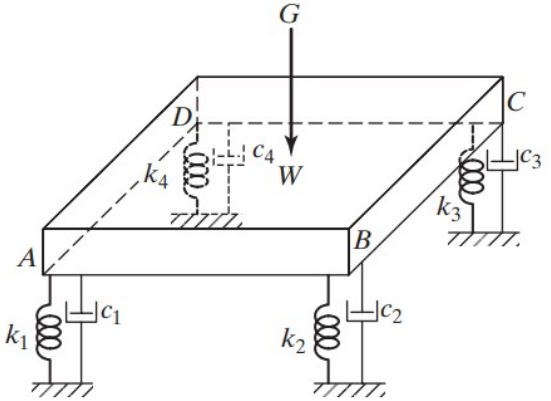
$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_p\left(\frac{\dot{x}}{r_p}\right)^2 + \frac{1}{2}\left(\frac{m_1 l_1^2}{3}\right)\left(\frac{\dot{x}}{r_p}\right)^2 + \frac{1}{2}m_2\left(\frac{\dot{x}l_1}{r_p}\right)^2 + \frac{1}{2}\left(\frac{m_c r_c^2}{2}\right)\left(\frac{\dot{x}l_1}{r_p r_c}\right)^2 + \frac{1}{2}m_c\left(\frac{\dot{x}l_1}{r_p}\right)^2$$

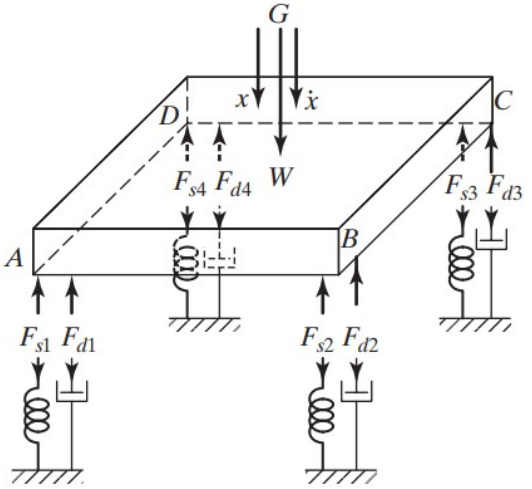
$$T = \frac{1}{2}m_{\text{eq}}\dot{x}^2 \qquad m_{\text{eq}} = m + \frac{J_p}{r_p^2} + \frac{1}{3}\frac{m_1 l_1^2}{r_p^2} + \frac{m_2 l_1^2}{r_p^2} + \frac{1}{2}\frac{m_c l_1^2}{r_p^2} + m_c\frac{l_1^2}{r_p^2}$$

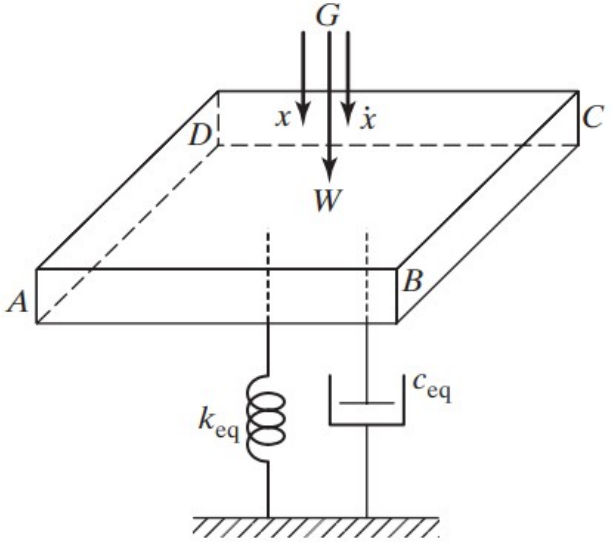
$$T = \frac{1}{2} m_p \dot{x}_p^2 + \frac{1}{2} m_v \dot{x}_v^2 + \frac{1}{2} J_r \dot{\theta}_r^2 + \frac{1}{2} m_r \dot{x}_r^2$$

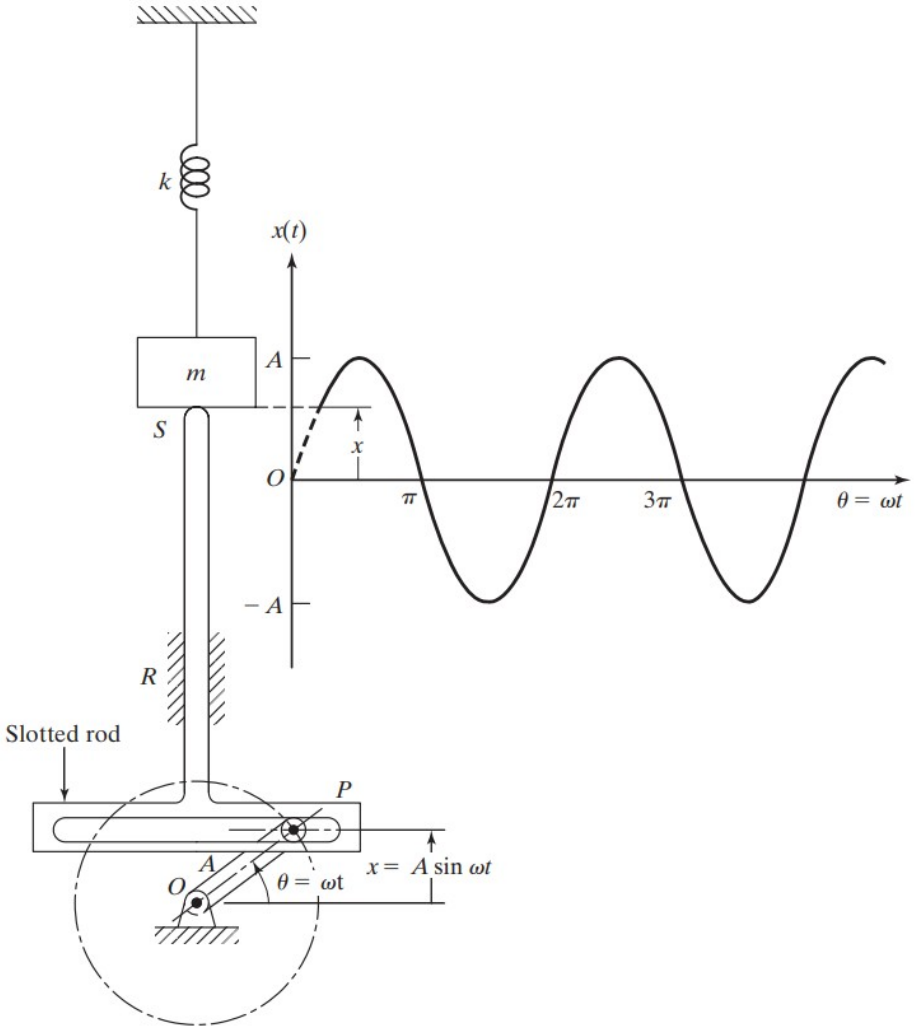



Overarm
Spindle
Cutter
Table
Knee
C
A
B
Base
Shock mounts
(at all four corners)
(a)


G
D
C
k_4
c_4
W
c_3
k_3
A
B
k_1
c_1
k_2
c_2
(b)


G
x
$\dot{x}$
D
C
F_{s4} F_{d4}
W
F_{s3} F_{d3}
A
B
F_{s1} F_{d1}
F_{s2} F_{d2}
(c)

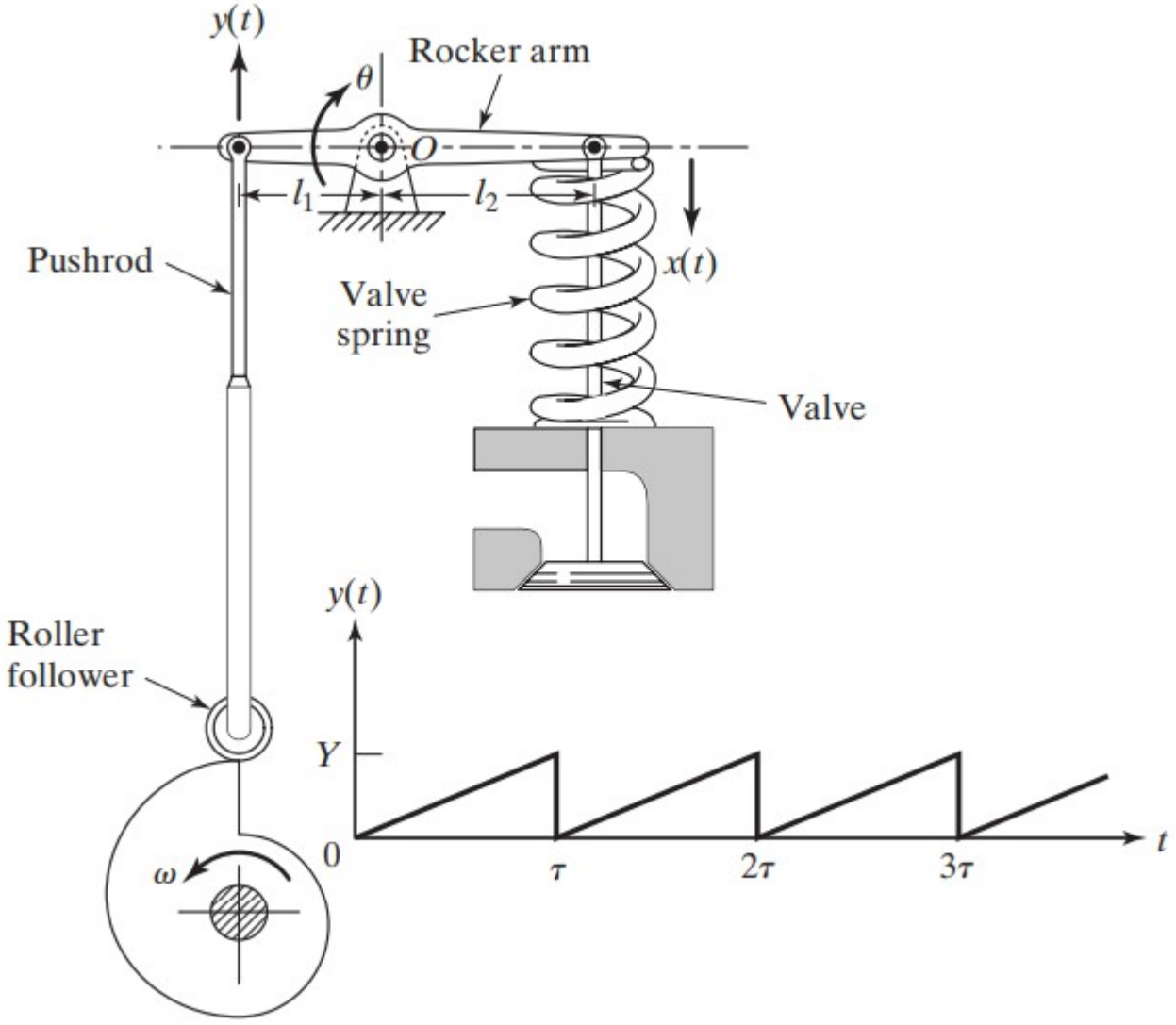

G
x
$\dot{x}$
D
C
W
A
B
k_{eq}
c_{eq}
(d)



$$x = A \sin \theta = A \sin \omega t$$

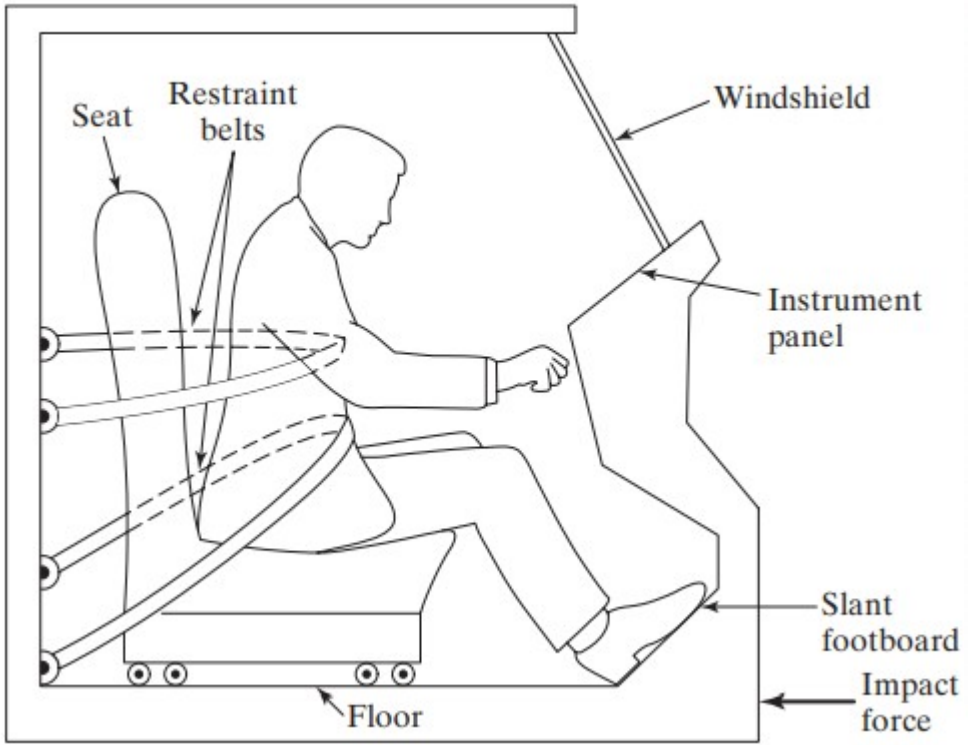
$$\frac{dx}{dt} = \omega A \cos \omega t$$

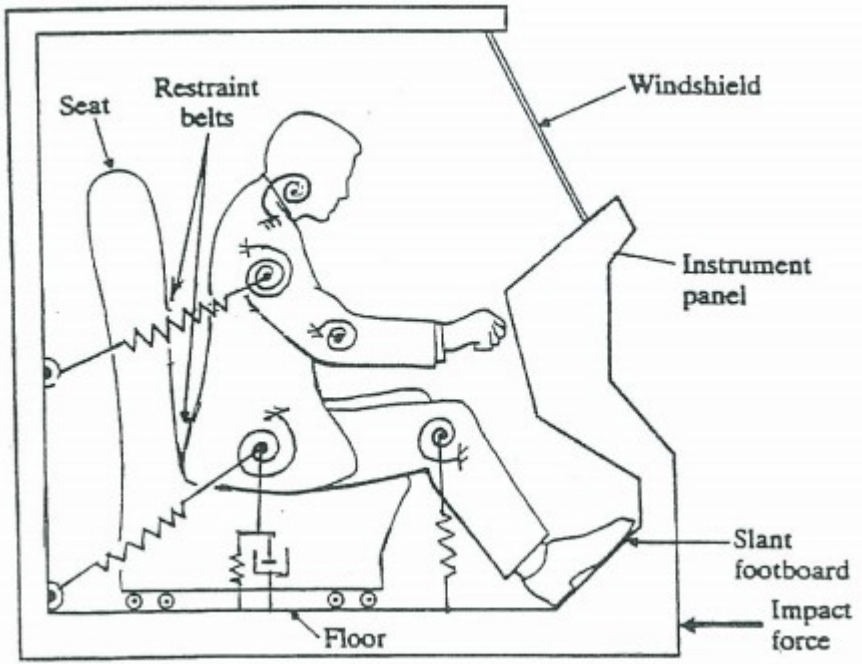
$$\frac{d^2x}{dt^2} = -\omega^2 A \sin \omega t = -\omega^2 x$$



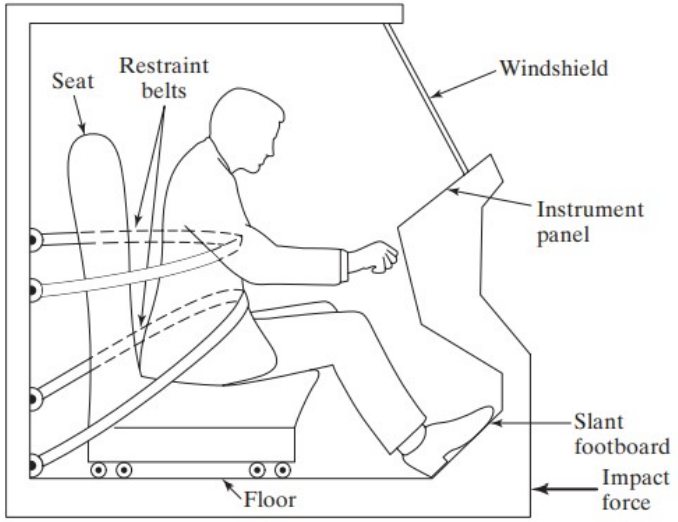
$$x(t) = A\frac{t}{\tau};$$

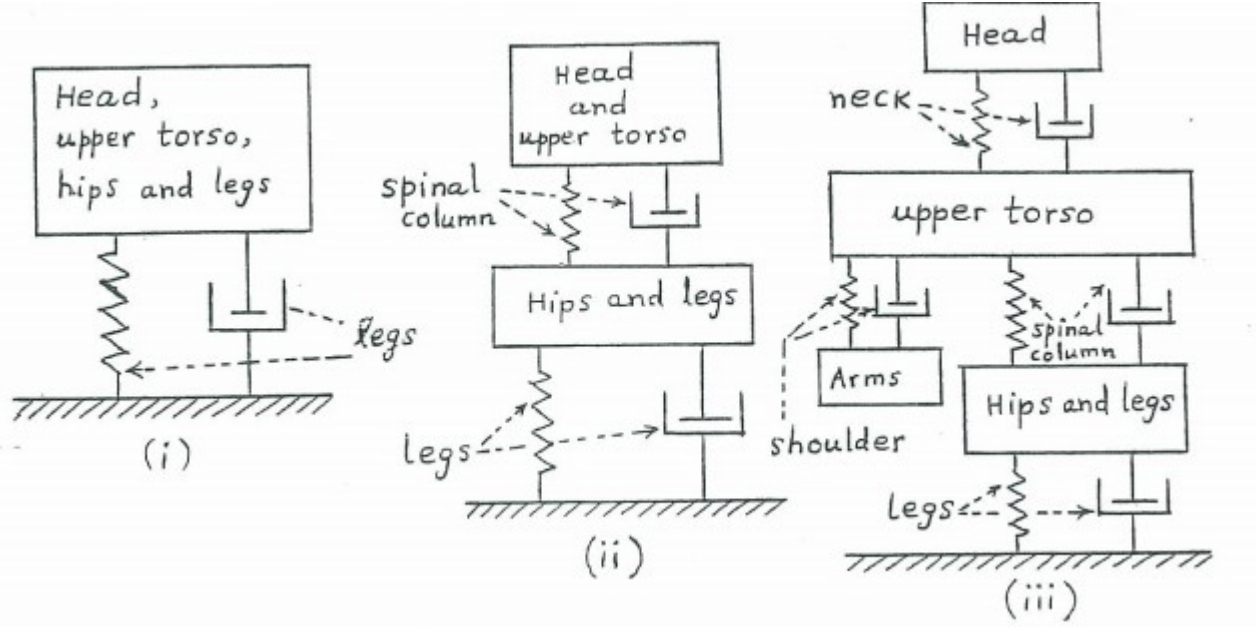
Projekt do samodzielnego wykonania



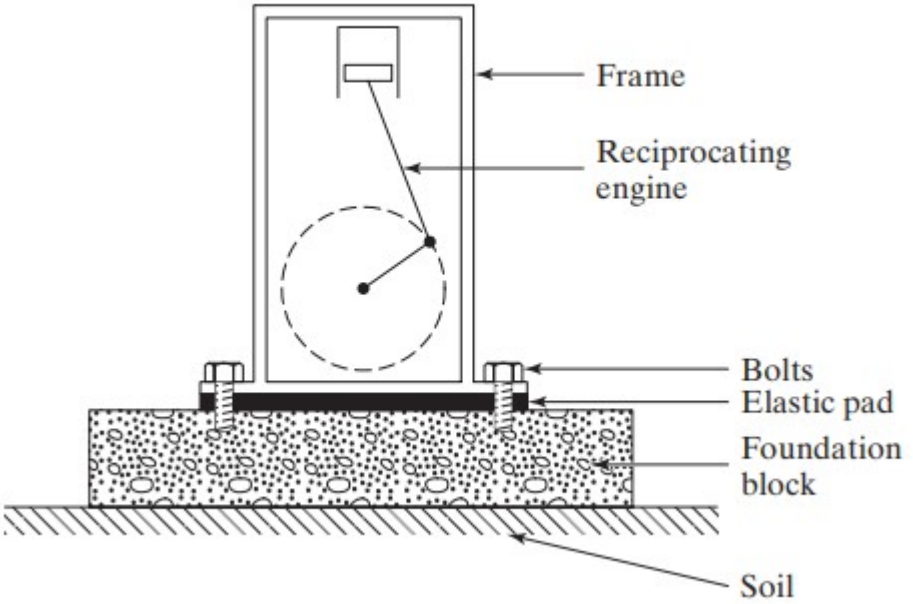


Projekt do samodzielnego wykonania

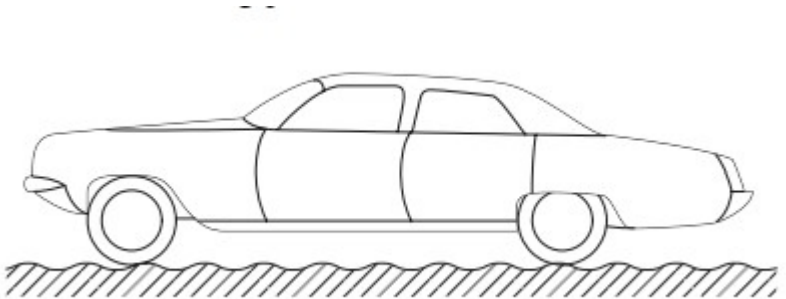




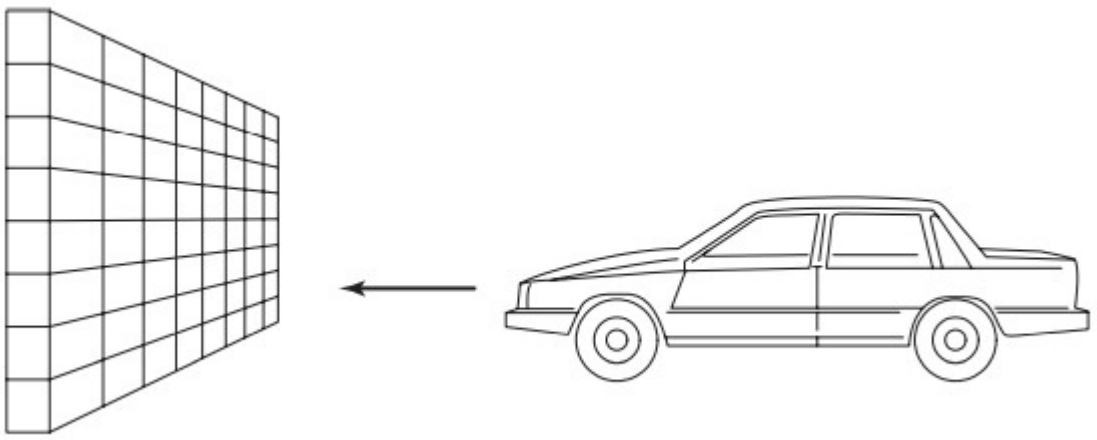
Projekt 1 zbudować schemat zastępczy



Projekt 1 zbudować schemat zastępczy



Projekt 1 zbudować schemat zastępczy



Projekt 1 zbudować schemat zastępczy