Projekt 19.03.2021

Przykład

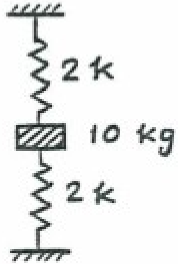
Sprężyna śrubowa zamocowana z jednej strony jest obciążona siła 100N co powoduje jej wydłużenie o 10 mm. Końce pionowej sprężyny są zamocowane przegubowo w połowie jej długości umieszczono masę m=10kg.Określić częstość drgań własnych układu

$$k = 100 \Big/ \left(\frac{10}{1000}\right) = 10000 \ N/m$$

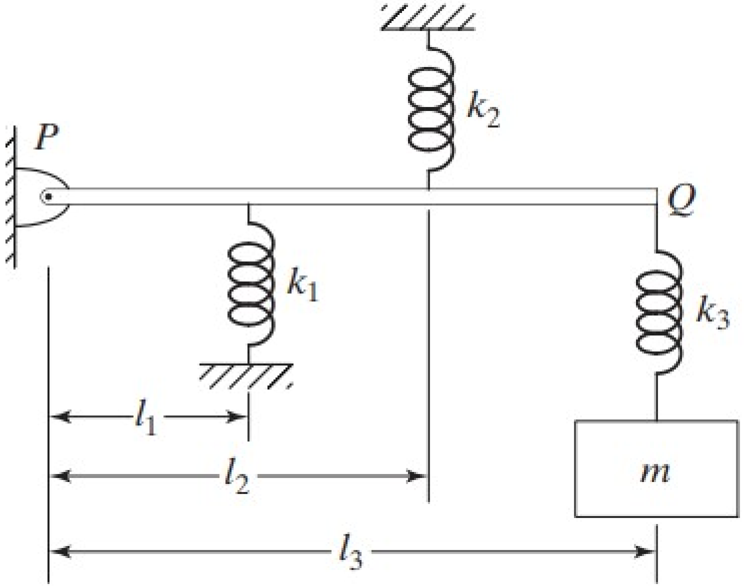
$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{4k}{m}} = \left(\frac{4 \times 10^4}{10}\right)^{1/2}$$

$$= 63 \cdot 2456 \ rad/sec$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{6 \cdot 2832}{63 \cdot 2456} = 0 \cdot 0993 \ sec$$



Trzy sprężyny i masa m są przymocowane do pręta PQ. Znaleźć częstość drgań własnych układu



For small angular rotation of bar PQ about P,

$$\frac{1}{2}(k_{12})_{eq}(\theta l_3)^2 = \frac{1}{2} k_1 (\theta l_1)^2 + \frac{1}{2} k_2 (\theta l_2)^2$$

i.e., $$(k_{12})_{eq} = \left(k_1 l_1^2 + k_2 l_2^2\right) / l_3^2$$

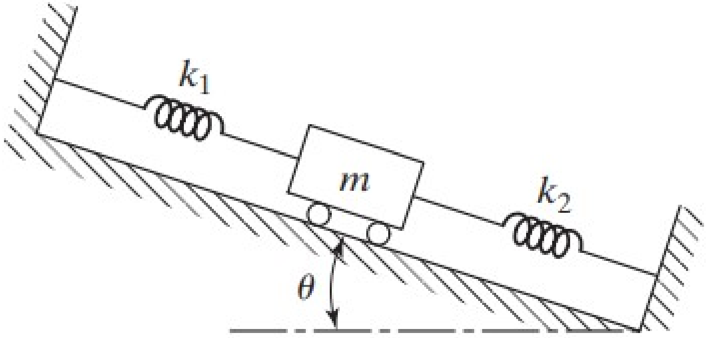
Let k_{eq} = overall spring constant at Q.

$$\frac{1}{k_{eq}} = \frac{1}{(k_{12})_{eq}} + \frac{1}{k_3}$$

$$k_{eq} = \frac{(k_{12})_{eq}\, k_3}{(k_{12})_{eq} + k_3} = \frac{\left\{k_1\left(\frac{l_1}{l_3}\right)^2 + k_2\left(\frac{l_2}{l_3}\right)^2\right\} k_3}{k_1\left(\frac{l_1}{l_3}\right)^2 + k_2\left(\frac{l_2}{l_3}\right)^2 + k_3}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1 k_2 l_1^2 + k_2 k_3 l_2^2}{m\left(k_1 l_1^2 + k_2 l_2^2 + k_3 l_3^2\right)}}$$

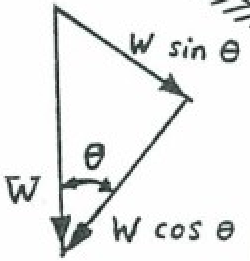
Znaleźć częstotliwości drgań własnych układu pokazanego na rysunku

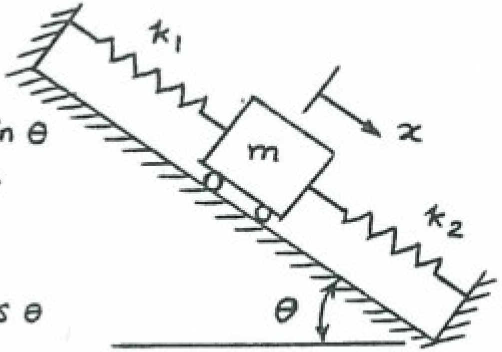


Let x be measured from the position of mass at which the springs are unstretched.

Equation of motion is

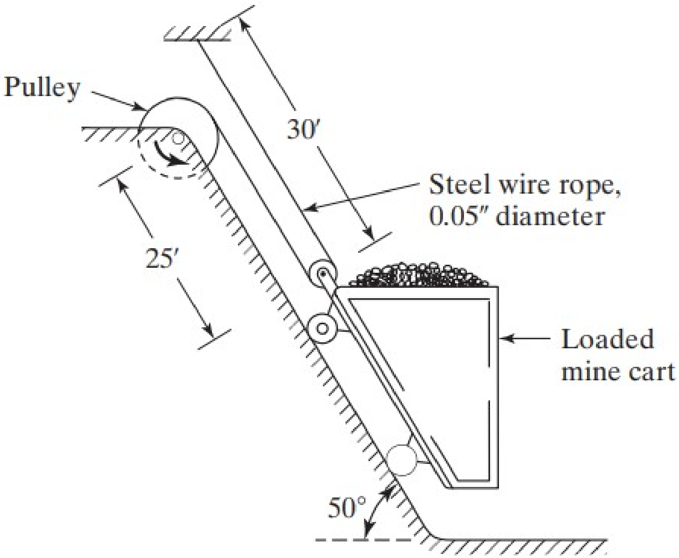
$$m\ddot{x} = -k_1(x+\delta_{st}) - k_2(x+\delta_{st}) + W\sin\theta \quad ---- \quad (E_1)$$





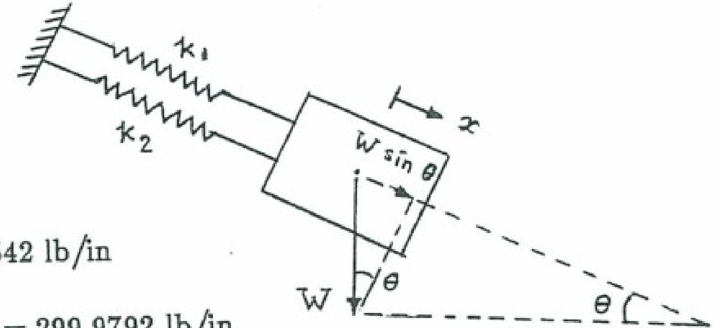
where $\delta_{st}(k_1+k_2) = W\sin\theta$.

Thus Eq. (E_1) becomes $m\ddot{x} + (k_1+k_2)\,x = 0 \Rightarrow \omega_n = \sqrt{\dfrac{k_1+k_2}{m}}$.



$$k_1 = \frac{A_1 E_1}{\ell_1} = \frac{\frac{\pi}{4}(0.05)^2 (30 (10^6))}{30 (12)}$$

$$= 163.6250 \text{ lb/in}$$



$$k_2 = \frac{A_2 E_2}{\ell_2} = \frac{163.625 (25)}{30} = 136.3542 \text{ lb/in}$$

$$k_{eq} = k_1 + k_2 = 163.6250 + 136.3542 = 299.9792 \text{ lb/in}$$

Let x be measured from the unstretched length of the springs. The equation of motion is:

$$m \ddot{x} = -(k_1 + k_2)(x + \delta_{st}) + W \sin\theta$$

where $(k_1 + k_2)\,\delta_{st} = W \sin\theta$

$$\text{i.e.,}\quad m \ddot{x} + (k_1 + k_2)\, x = 0$$

Thus the natural frequency of vibration of the cart is given by

$$\omega_n = \sqrt{\frac{k_1 + k_2}{m}} = \sqrt{\frac{299.9792 (386.4)}{5000}} = 4.8148 \text{ rad/sec}$$