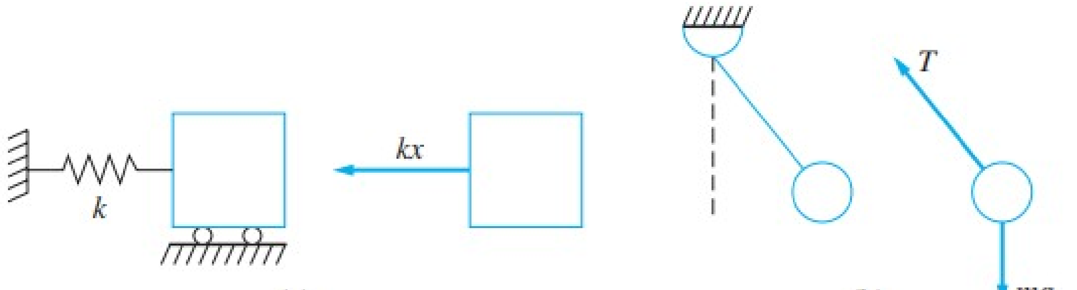
Drgania w konstrukcjach lotniczych

podstawy

Przykłady prostych układów drgających



(a) (b)

(a) (b)

Modelowanie matematyczne w drganiach

1. Identyfikacja obiektu analizy
2. Założenia upraszczające
3. Podstawowe prawa mechaniki
4. Związki konstytutywne
5. Wiązania geometryczne
6. Schematy
7. Rozwiązania matematyczne
8. Fizyczna interpretacja

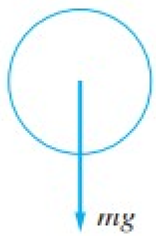
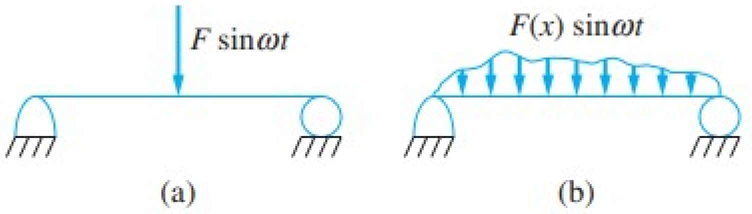
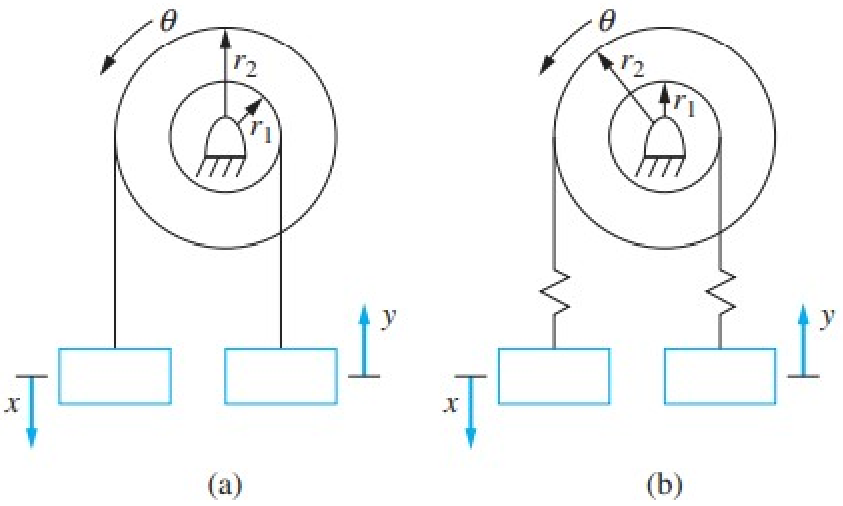


FIGURE 1.3



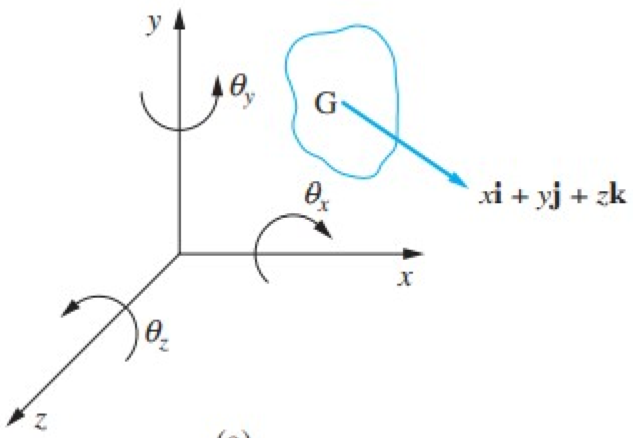
Układy współrzędnych



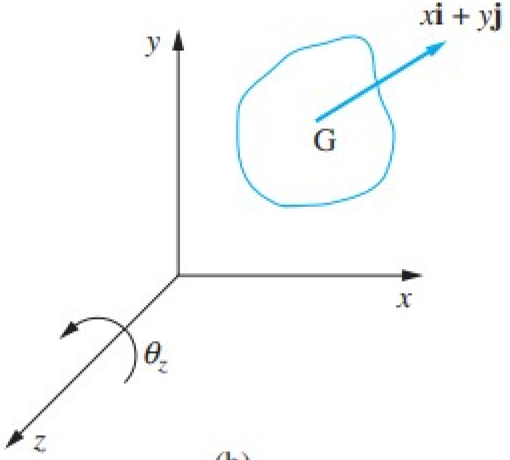
$$x = r_2\theta \qquad (1.1)$$

$$y = r_1\theta = \frac{r_1}{r_2} \qquad (1.2)$$

Układy współrzędnych



(a)



(b)

Określanie liczby stopni swobody (DOF)

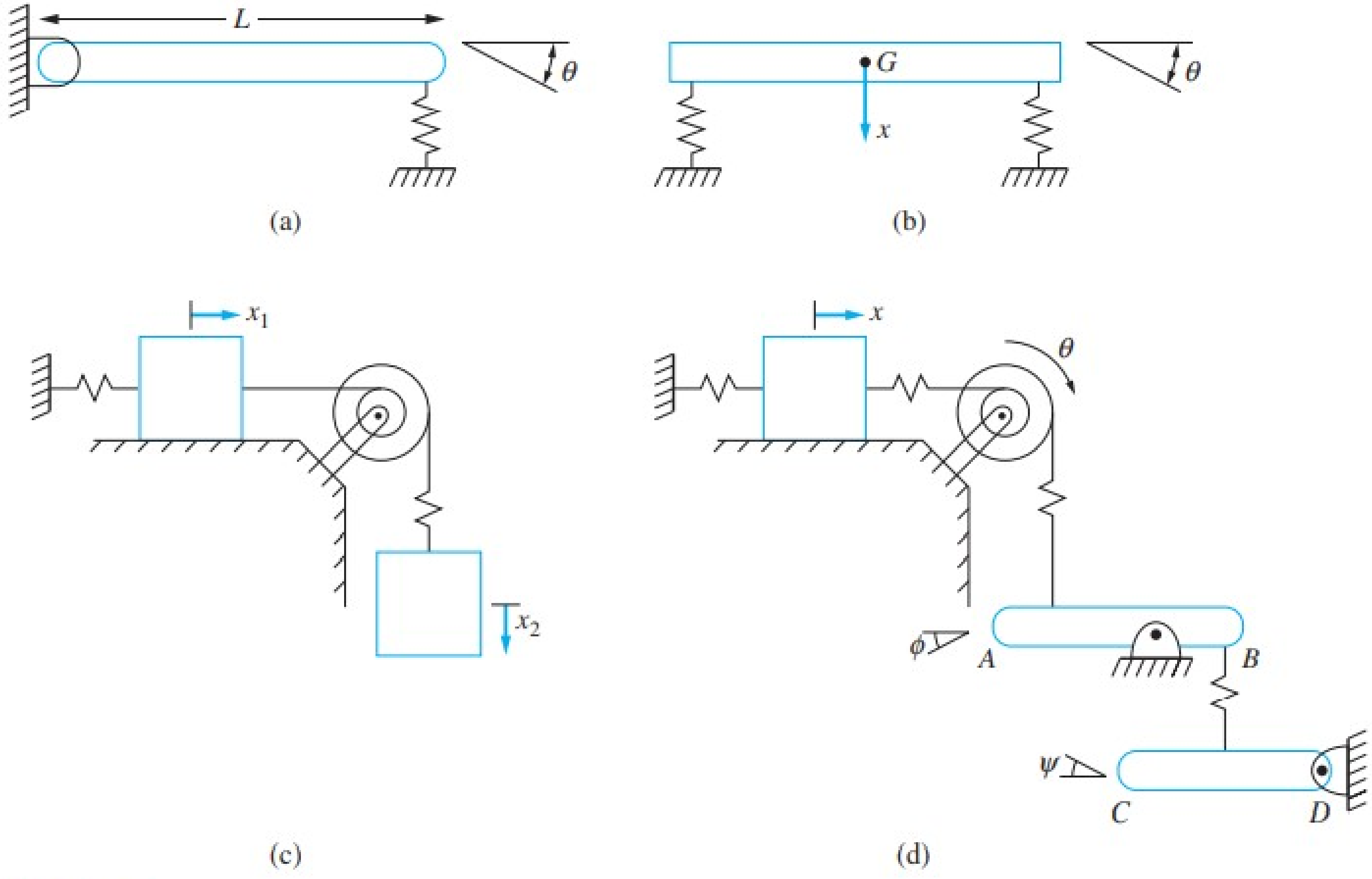
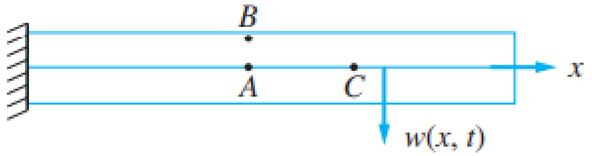


FIGURE 1.7

1.2.2.4

Stopnie swobody (DOF) układów ciągłych



Klasyfikacja drgań

- Drgania układów o jednym (SDOF) i wielu stopniach (MDOF)
- Drgania swobodne (free vibrations), drgania wymuszone (forced vibration)
- Drgania harmoniczne, losowe(random)
- Drgania tłumione (damped), nietłumione (undamped)
- Drgania liniowe i nieliniowe

Analiza wymiarowa

- Funkcja celu y=f(x1,x2,x3,x4) gdzie x1,x2,x3,x4 zmienne niezależne
- Siła oporu w mechanice płynów $D = f(v, L, \rho, \mu, c)$

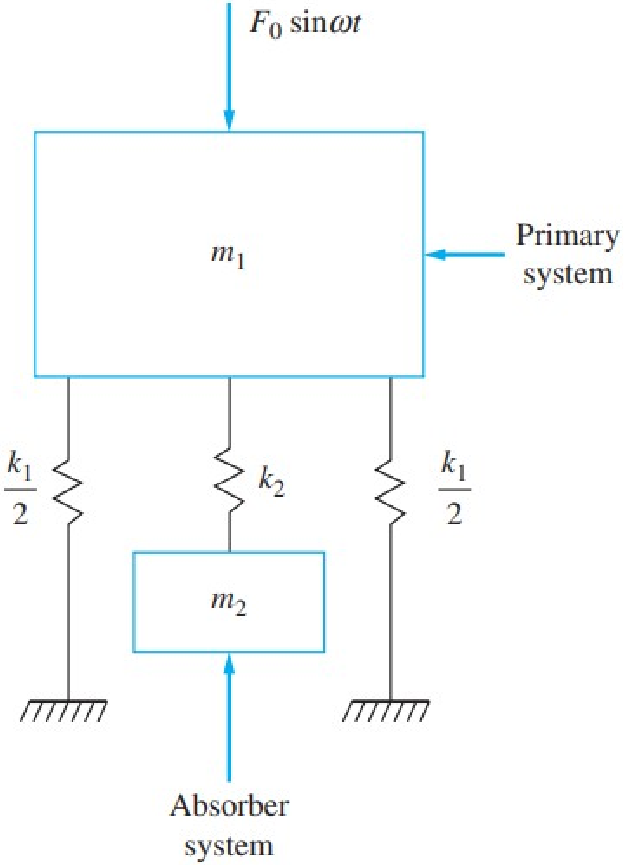
$$C_D = f(Re, M)$$

$$C_D = \frac{D}{\frac{1}{2}\rho v^2 L}$$

$$Re = \frac{\rho v L}{\mu}$$

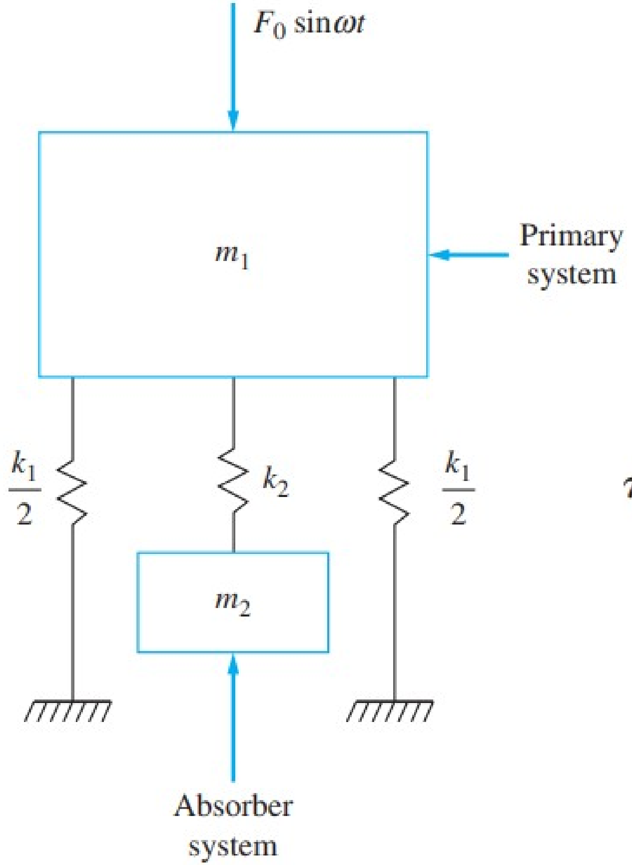
$$M = \frac{v}{c}$$

Analiza wymiarowa, problem



$$X_1 = F_0 \left| \frac{k_2 - m_2\omega^2}{m_1 m_2 \omega^2 - (k_2 m_1 + k_1 m_2 + k_2 m_2)\omega^2 + k_1 k_2} \right|$$

Analiza wymiarowa, redukcja ilości wymiarów



$$X_1 = \frac{F_0}{k_1}\left|\frac{1 - \dfrac{m_2\omega^2}{k_2}}{\dfrac{m_1 m_2 \omega^4}{k_1 k_2} - \left(\dfrac{m_1}{k_1} + \dfrac{m_2}{k_2} + \dfrac{m_2}{k_1}\right)\omega^2 + 1}\right|$$

Mnożymy $\dfrac{k_1}{F_0}$ i podstawiamy $\pi_1 = \dfrac{k_1 x_1}{F_0}$ $\pi_2 = \dfrac{m_2\omega^2}{k_2}$,

$$\pi_1 = \left|\frac{1 - \pi_2}{\dfrac{m_1\omega^2}{k_1}\pi_2 - \pi_2 + \left(\dfrac{m_1}{k_1} + \dfrac{m_2}{k_1}\right)\omega^2 + 1}\right|$$

ie $\pi_3 = \dfrac{m_1\omega^2}{k_1}$.

Wzór bezwymiarowy

$$\pi_1 = \left|\frac{1 - \pi_2}{\pi_3\pi_2 - \pi_2 + (1 + \pi_4)\,\pi_3 + 1}\right|$$

Prosty ruch harmoniczny

$$x(t) = A\cos\omega t + B\sin\omega t \qquad (1.10)$$

$$\sin(\omega t + \phi) = \sin\omega t\cos\phi + \cos\omega t\sin\phi \qquad (1.11)$$

$$x(t) = X\sin(\omega t + \phi) \qquad (1.12)$$

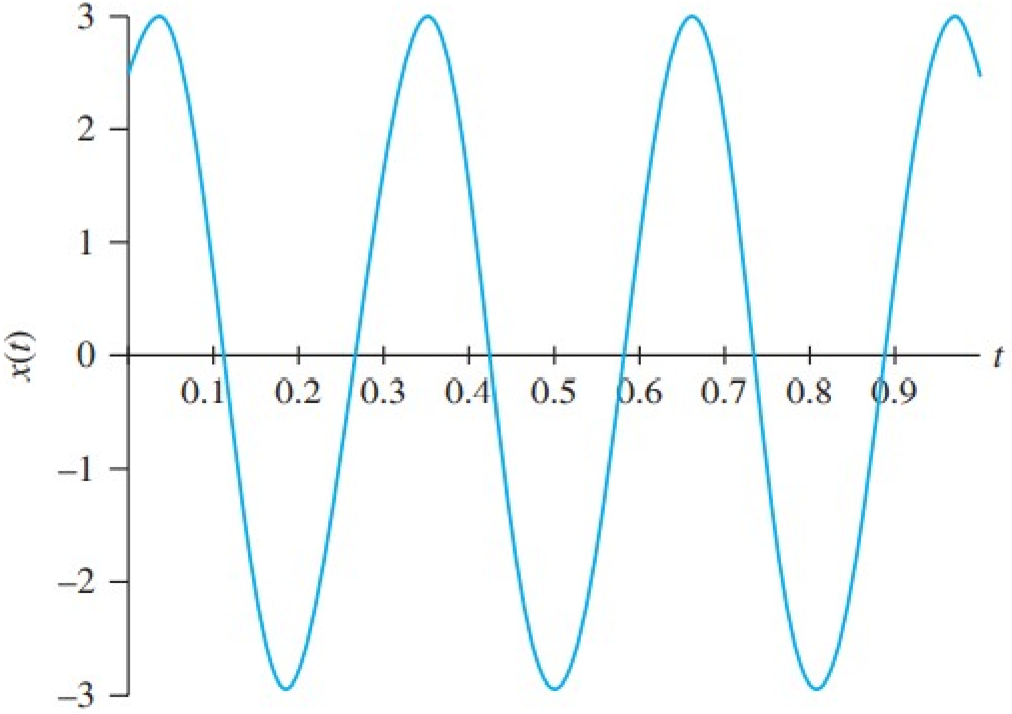
$$X = \sqrt{A^2 + B^2} \qquad (1.13)$$

$$\phi = \tan^{-1}\left(\frac{A}{B}\right) \qquad (1.14)$$

$$T = \frac{2\pi}{\omega} \qquad (1.15)$$

$$f = \frac{\omega}{2\pi} \qquad (1.16)$$

Prosty ruch harmoniczny



$$f = \left(\frac{\omega}{2\pi}\ \text{cycle/s}\right)(2\pi\ \text{rad/cycle}) = \omega\ \text{rad/s}$$

$$\omega\ \text{rpm/s} = (\omega\ \text{rad/s})\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)$$

Prosty ruch harmoniczny, przykład

$$x(t) = 0.003\ \cos(30t) + 0.004\ \sin(30t)\ \mathrm{m}$$

$$T = \frac{2\pi}{30}\ \mathrm{s} = 0.209\ \mathrm{s}$$

$$f = \frac{1}{T} = \frac{1}{0.209\ \mathrm{s}} = 4.77\ \mathrm{Hz}$$

$$\omega = 2\pi f = 30\ \mathrm{rad/s}$$

$$\omega = \left(20\frac{\mathrm{rad}}{\mathrm{s}}\right)\left(\frac{1\ \mathrm{rev}}{2\pi\ \mathrm{rad}}\right)\left(\frac{60\ \mathrm{s}}{1\ \mathrm{min}}\right) = 191.0\ \mathrm{rpm}$$

$$\phi = \tan^{-1}\left(\frac{0.003}{0.004}\right) = 0.643\ \mathrm{rad}$$

$$x(t) = 0.005\sin(30t + 0.643)\ \mathrm{m}$$

Podstawy mechaniki, kinematyka

$$\mathbf{r} = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k} \qquad (1.19)$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{x}(t)\mathbf{i} + \dot{y}(t)\mathbf{j} + \dot{z}(t)\mathbf{k} \qquad (1.20)$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \ddot{x}(t)\mathbf{i} + \ddot{y}(t)\mathbf{j} + \ddot{z}(t)\mathbf{k} \qquad (1.21)$$

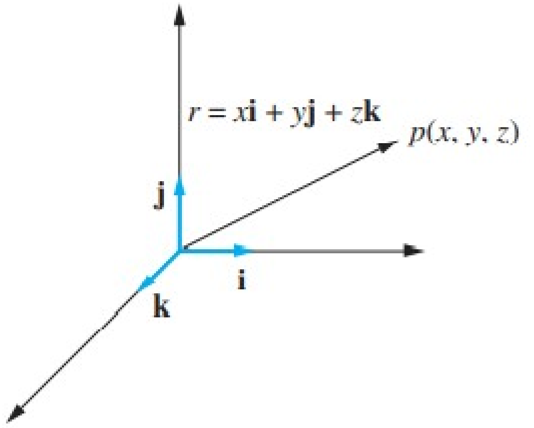
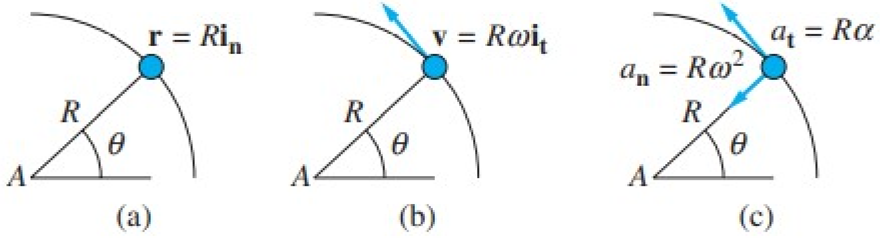
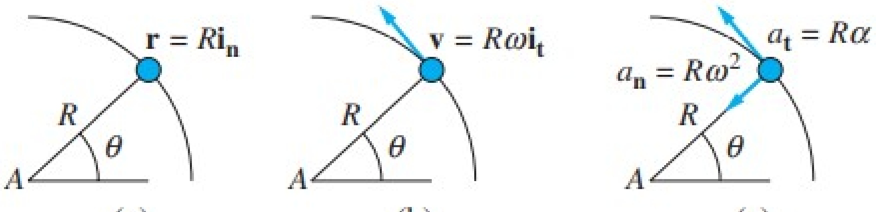


FIGURE 1.11



Podstawy mechaniki, kinematyka



(a) (b) (c)

$$\dot{\theta} = \omega \qquad (1.22)$$

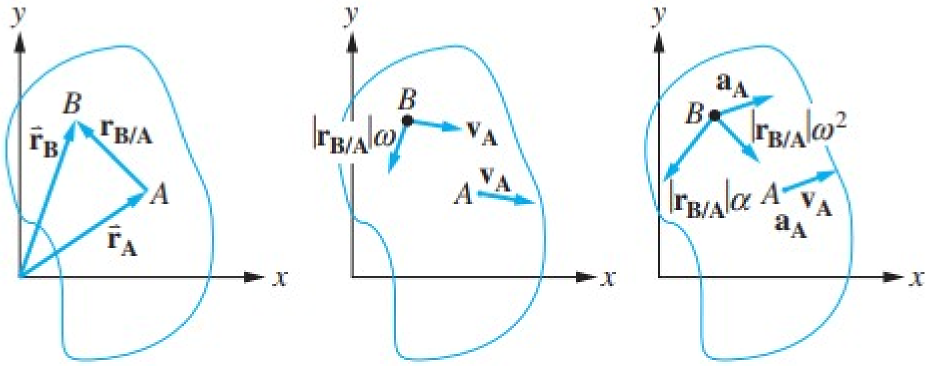
$$\alpha = \ddot{\theta} \quad \text{rad/s}^2. \qquad (1.23)$$

$$r = R\mathbf{i}_\mathrm{n} \qquad (1.24)$$

$$\frac{d\mathbf{i}_\mathrm{t}}{dt} = -\omega\mathbf{i}_\mathrm{n} \qquad \frac{d\mathbf{i}_\mathrm{n}}{dt} = -\omega\mathbf{i}_\mathrm{n}, \qquad \mathbf{v} = \dot{\mathbf{r}} = R\frac{d\mathbf{i}_\mathrm{n}}{dt} = R\omega\mathbf{i}_\mathrm{t} \qquad (1.25)$$

$$\mathbf{a} = \dot{\mathbf{v}} = \frac{d(R\omega\mathbf{i}_\mathrm{t})}{dt} = R\frac{d\omega}{dt}\mathbf{i}_\mathrm{t} + R\omega\frac{d\mathbf{i}_\mathrm{t}}{dt} = R\alpha\mathbf{i}_\mathrm{t} - R\omega^2\mathbf{i}_\mathrm{n} \qquad (1.26)$$

Podstawy mechaniki, kinematyka



(a) (b) (c)

$$\mathbf{r_B} = \mathbf{r_A} + \mathbf{r_{B/A}} \quad (1.27)$$

$$\mathbf{v_B} = \mathbf{v_A} + \mathbf{v_{B/A}} \quad (1.28)$$

$$v_{B/A} = |\mathbf{r_{B/A}}|\omega \quad (1.29)$$

$$\mathbf{a_B} = \mathbf{a_A} + \mathbf{a_{B/A}} \quad (1.30)$$

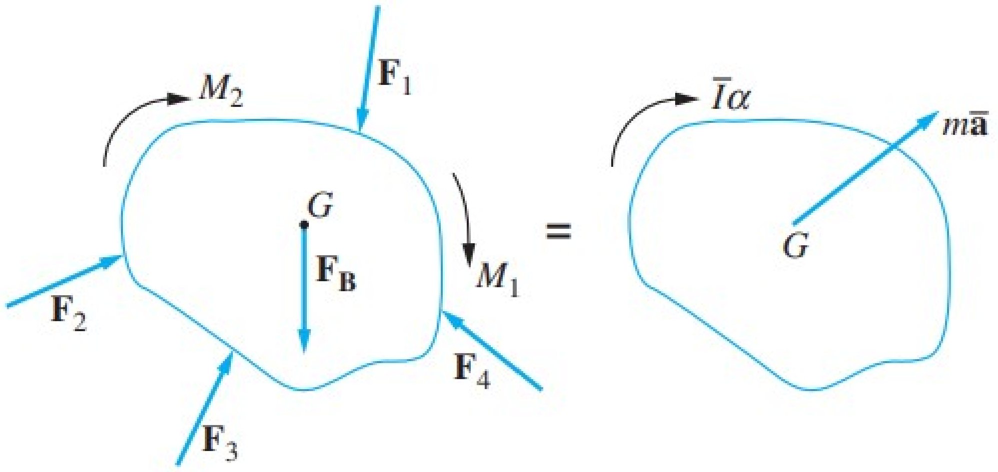
$$\mathbf{a_B} = |\mathbf{r_{B/A}}|\alpha\mathbf{i_t} - r\omega^2\mathbf{i_n} \quad (1.31)$$

Podstawy mechaniki , dynamika

$$\sum \mathbf{F} = m\mathbf{a} \qquad (1.32)$$

$$\sum \mathbf{F} = m\bar{\mathbf{a}} \qquad (1.33)$$

$$\sum M_G = \bar{I}\alpha \qquad (1.34)$$

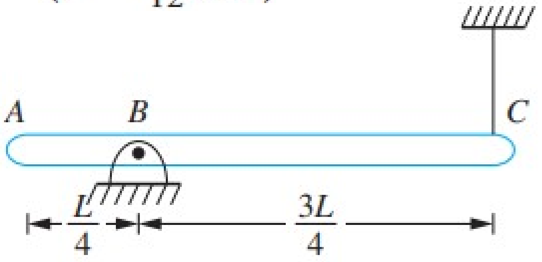


$$\sum \mathbf{F}_{ext} = \sum \mathbf{F}_{eff} \qquad (1.36)$$

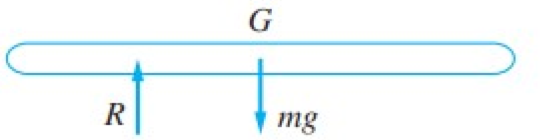
$$\sum M_{O_{ext}} = \sum M_{O_{eff}} \qquad (1.37)$$

Przykład, znaleźć przyśpieszenie kątowe

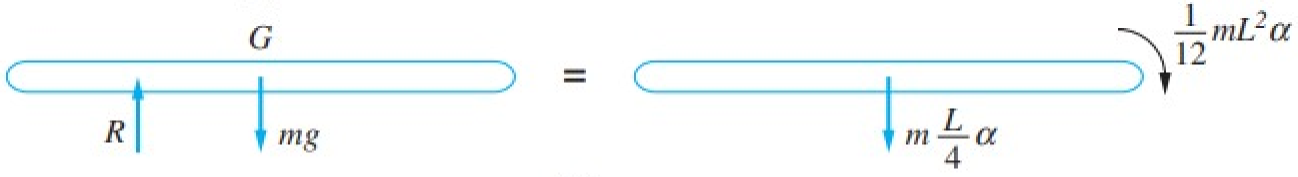
$$(\bar{I} = \frac{1}{12} mL^2)$$



(a)



(b)



(c)

$$\sum M_B = \sum I_B \alpha \qquad \text{(a)}$$

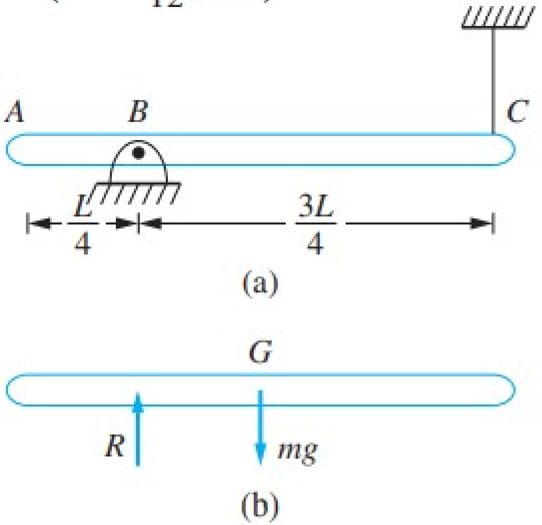
$$mg \frac{L}{4} = I_B \alpha \qquad \text{(b)}$$

$$I_B = \bar{I} + md^2 = \frac{1}{12} mL^2 + m\left(\frac{L}{4}\right)^2 = \frac{7}{48} mL^2 \qquad \text{(c)}$$

$$\alpha = \frac{12g}{7L} \qquad \text{(d)}$$

Przykład, znaleźć przyśpieszenie kątowe

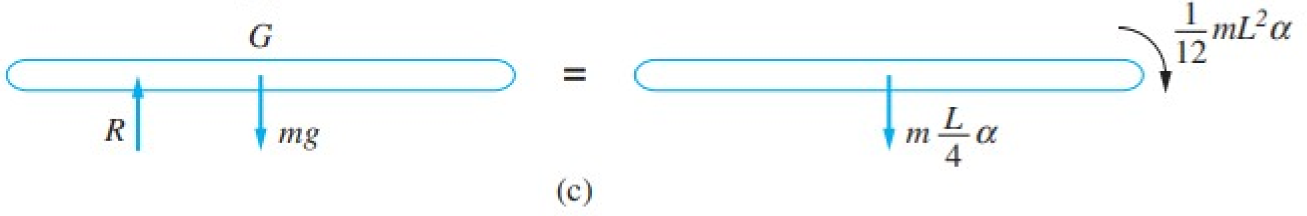
$(\bar{I} = \frac{1}{12} mL^2)$



$$(\sum M_B)_{ext} = (\sum M_B)_{eff}$$

$$mg\frac{L}{4} = \frac{1}{12} mL^2 + \left(m\frac{L}{4}\alpha\right)\left(\frac{L}{4}\right)$$

$$\alpha = \frac{12g}{7L}.$$



Znaleźć kątowe przyśpieszenie

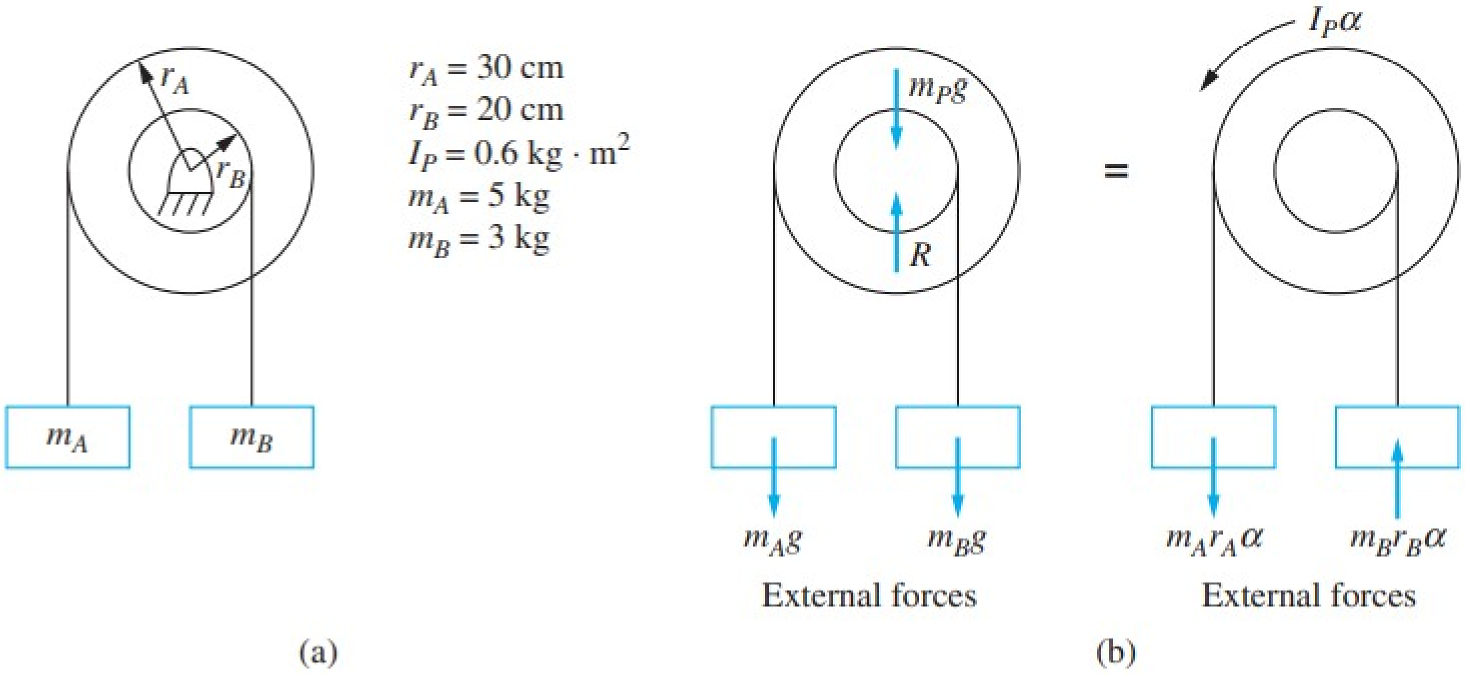


FIGURE 1.17

$$\sum M_{O_{ext}} = \sum M_{O_{eff}}$$

$$m_A g r_A - m_B g r_B = I_p \alpha + m_B r_A^2 \alpha + m_B r_B^2 \alpha \longrightarrow \alpha = 7.55 \text{ rad/s}^2.$$

Energia kinetyczna T, praca U

Ruch dowolny

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 \qquad (1.38)$$

Ruch obrotowy

$$T = I_O\omega^2 \qquad (1.39)$$

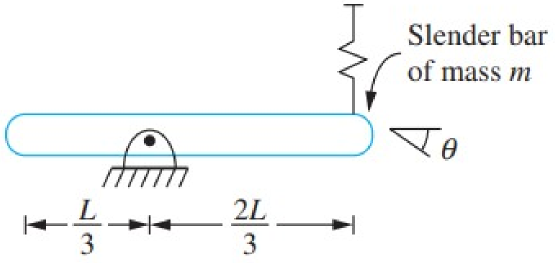
Praca siły F dla dwóch pozycji wektora wodzącego r

$$U_{A\to B} = \int_{\mathbf{r}_A}^{\mathbf{r}_B} \mathbf{F} \cdot d\mathbf{r} \qquad (1.40)$$

Praca momentu M F dla dwóch obrotu

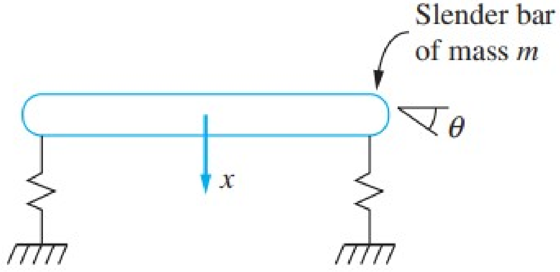
$$U_{A\to B} = \int_{\theta_A}^{\theta_B} M\, d\theta \qquad (1.41)$$

Przykład. Znaleźć energię



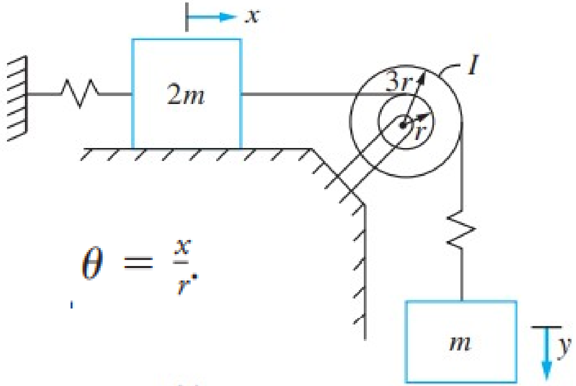
(a)

$$T = \frac{1}{2}m\left(\frac{L}{6}\dot{\theta}\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\dot{\theta}^2 = \frac{1}{18}mL^2\dot{\theta}^2$$



(b)

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\dot{\theta}^2$$

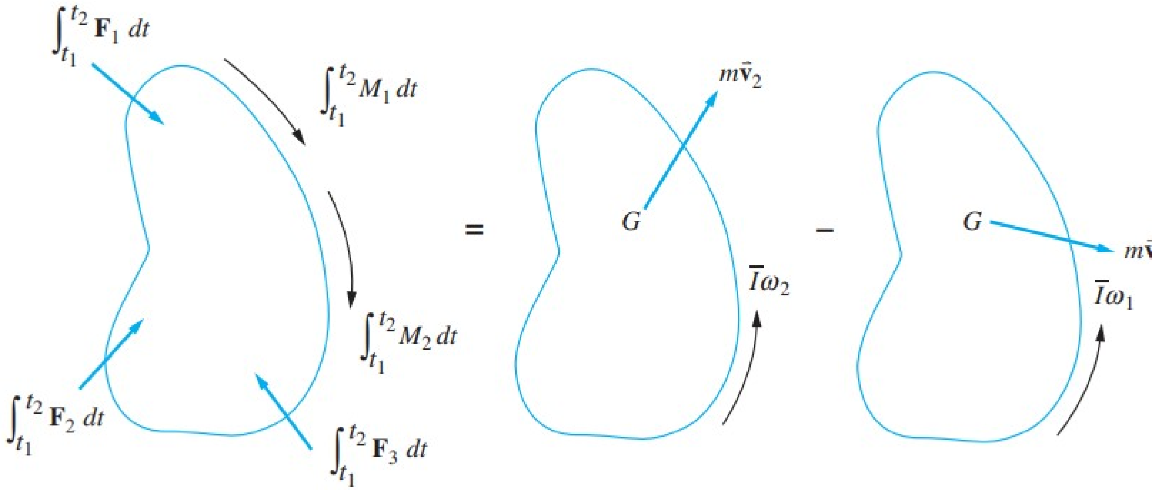


(c)

$$\theta = \frac{x}{r}.$$

$$T = \frac{1}{2}(2m)\dot{x}^2 + \frac{1}{2}I\left(\frac{\dot{x}}{r}\right)^2 + \frac{1}{2}m\dot{y}^2 = \frac{1}{2}\left(2m + \frac{I}{r^2}\right)\dot{x}^2 + \frac{1}{2}m\dot{y}^2$$

Pęd, moment pędu (kręt)

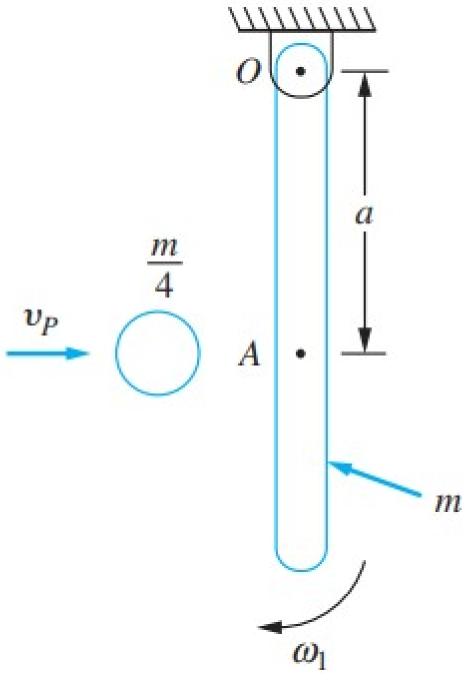


$$I_{1\rightarrow 2} = \int_{t_1}^{t_2} \mathbf{F} dt \quad (1.48)$$

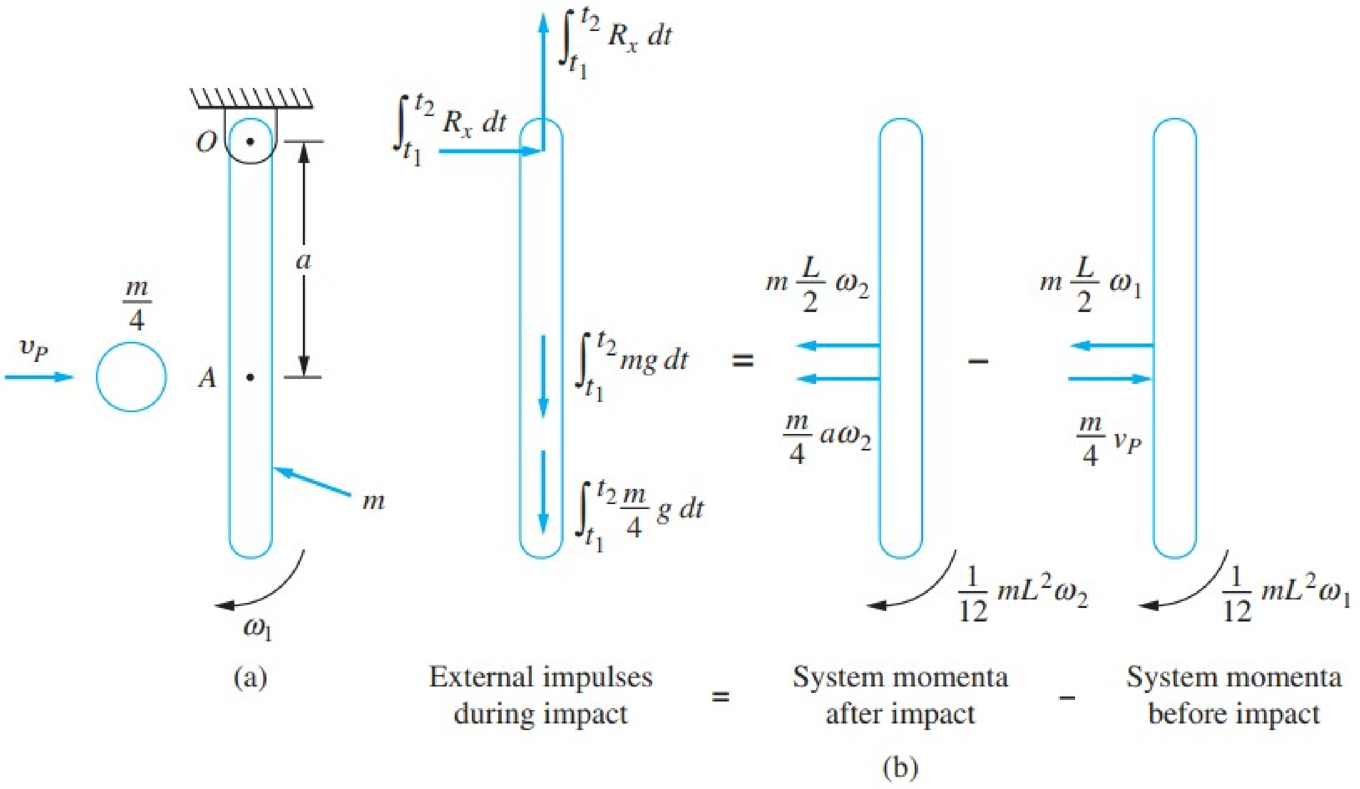
$$J_{O_{1\rightarrow 2}} = \int_{t_1}^{t_2} \sum M_O\, dt \quad (1.49)$$

$$L = m\bar{\mathbf{v}} \quad (1.50)$$

$$H_G = \bar{I}\omega \quad (1.51)$$



Pręt o masie m obraca się z prędkością kątową ω_1
w pręt uderza masa m/4 z prędkością v_p, całość się skleja.
Określić:
a) prędkość kątową tego układu,
b) maksymalny kąt wychylenia
c) przyspieszenie kątowe układu



$$0 = \left(m\frac{L}{2}\omega_2\right)\left(\frac{L}{2}\right) + \left(\frac{m}{4}a\omega_2\right)(a) + \frac{1}{12}mL^2\omega_2 - \left[\left(m\frac{L}{2}\omega_1\right)\left(\frac{L}{2}\right) - \left(\frac{m}{4}v_p\right)(a) + \frac{1}{12}mL^2\omega_1\right]$$

$$\longrightarrow \quad \omega_2 = \frac{4L^2\omega_1 - 3v_p a}{4L^2 + 3a^2}$$