Drgania w konstrukcjach lotniczych

podstawy

Modelowanie układów o jednym stopniu swobody

Sprężyna

Sprężyna liniowa $F = kx$

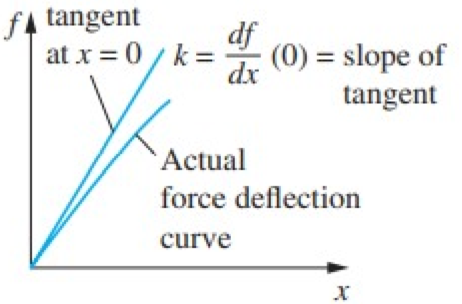
Praca przy odkształcaniu sprężyny $U_{1\to 2} = \int_{x_1}^{x_2} (-kx)dx = k\frac{x_1^2}{2} - k\frac{x_2^2}{2}$

$$F = f(x) \qquad (2.1)$$

Energia w sprężynie $V(x) = \frac{1}{2}kx^2$

$$F = k_0 + k_1 x + k_2 x^2 + k_3 x^3 + \cdots$$

Sprężyna skręcana



$$M = k_t\theta$$

$$V = \frac{1}{2}k_t\theta^2$$

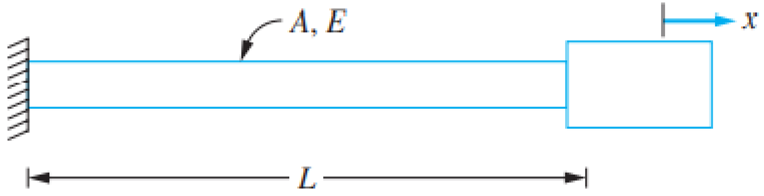
$$F = kx$$

$$\tau_{\max} = \frac{FrD}{2J} = \frac{16Fr}{\pi D^3}$$

$$J = (\pi D^4)/32$$

$$x = \frac{64Fr^3N}{GD^4}$$

$$k = \frac{GD^4}{64Nr^3}$$



$$\boldsymbol{\varepsilon} = \frac{F}{AE} = \frac{x}{L}$$

$$s = \frac{1}{2}\sigma\boldsymbol{\varepsilon} = \frac{1}{2}E\boldsymbol{\varepsilon}^2$$

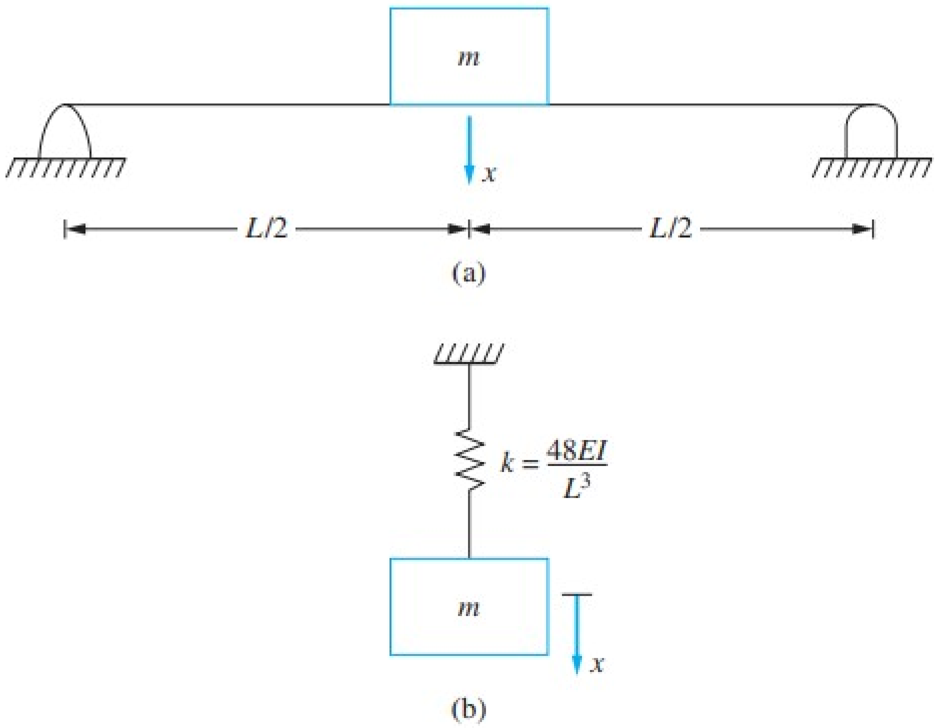
$$S = sV = \frac{1}{2}E\boldsymbol{\varepsilon}^2 AL = \frac{1}{2}(EA/L)x^2$$

$$F = \frac{AE}{L}x$$

$$k = \frac{AE}{L}$$

$$x = \frac{L^3}{48EI}F$$

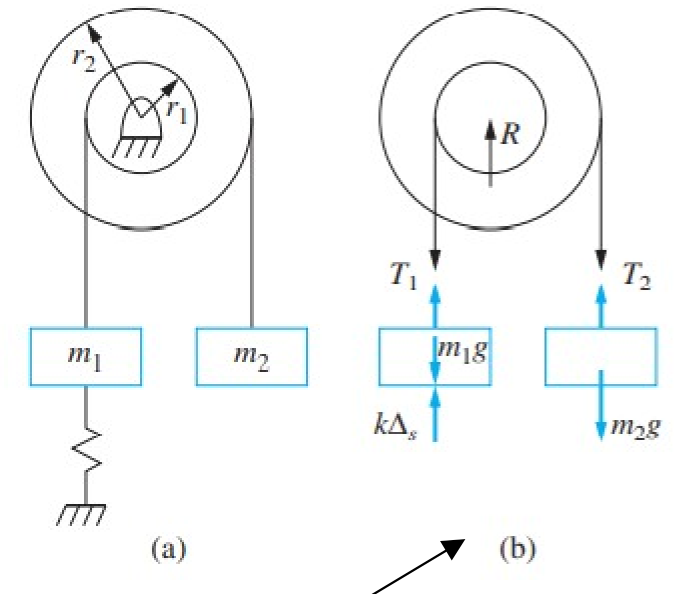
Układ zastępczy



Określmy deformację statyczną

Układ wyjściowy

Równania równowagi



$$T_1 = m_1 g - k\Delta_s$$

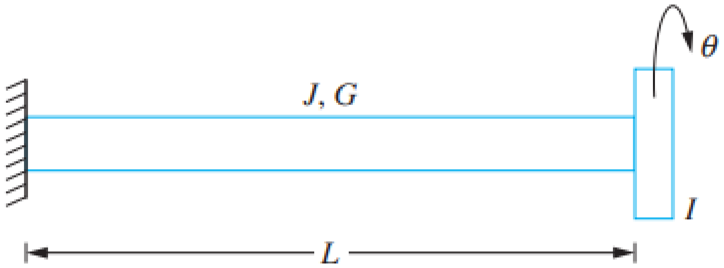
$$m_2 g r_2 - (m_1 g - k\Delta_s) r_1 = 0$$

$$\Delta_s = \frac{m_1 g r_1 - m_2 g r_2}{k r_1}$$

Statyczna deformacja

Układ zastępczy

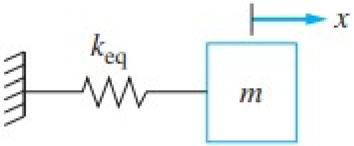
Określmy deformację statyczną



$$M = \frac{JG}{L}\theta$$

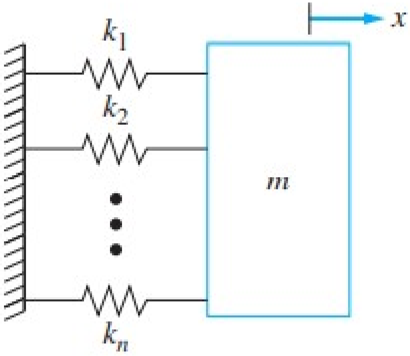
$$k_t = \frac{JG}{L}$$

Układy sprężyn



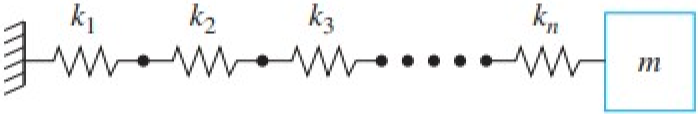
$$F = k_{eq} x$$

Układ równoległy



$$F = k_1 x + k_2 x + \cdots + k_n x = \left(\sum_{i=1}^{n} k_i \right) x$$

$$k_{eq} = \sum_{i=1}^{n} k_i$$



$$x = x_1 + x_2 + \cdots + x_n = \sum_{i=1}^{n} x_i \qquad x_i = F/k$$

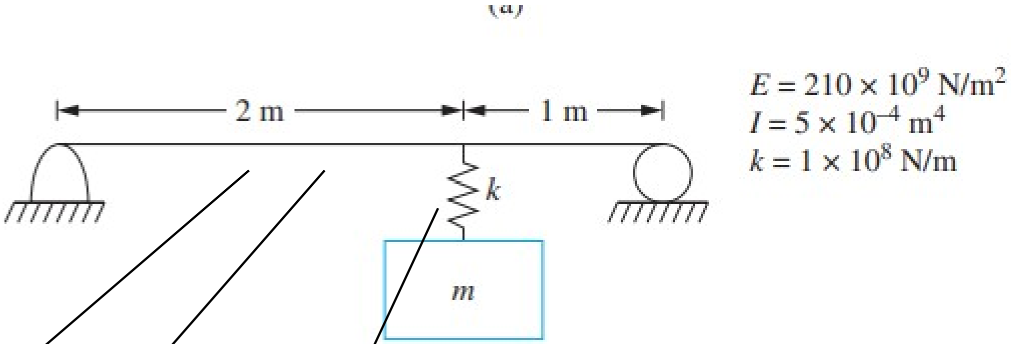
Układ szeregowy

$$x = \sum_{i=1}^{n} \frac{F}{k_i}$$

$$k_{eq} = \frac{1}{\sum_{i=1}^{n} \frac{1}{k_i}}$$

Układy sprężyn

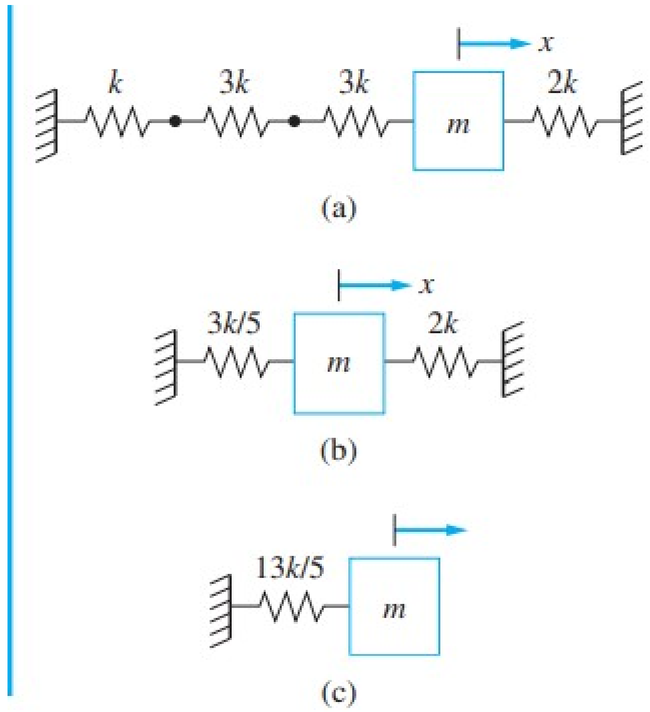
$$k_{eq} = \frac{1}{\sum_{i=1}^{n} \frac{1}{k_i}}$$

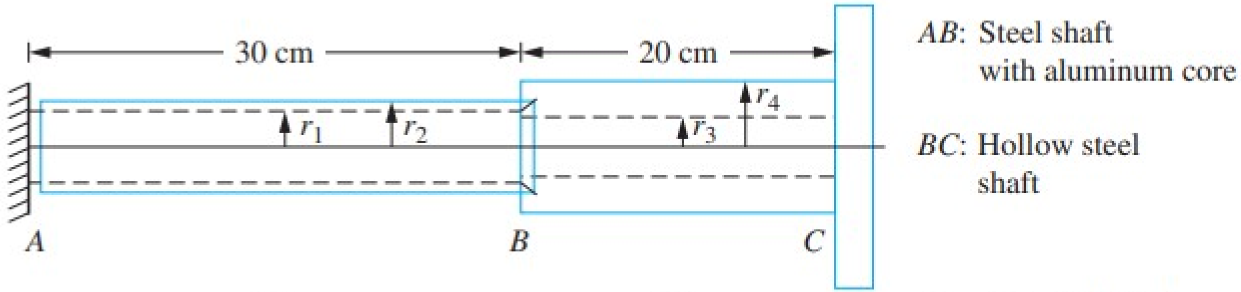


$$\omega(z = 2\text{ m}) = \omega\left(\frac{2L}{3}\right) = \frac{4L^3}{243EI}$$

$$k_{eq} = \frac{1}{\frac{1}{2.36 \times 10^8\text{ N/m}} + \frac{1}{1 \times 10^8\text{ N/m}}} = 7.03 \times 10^7\text{ N/m}$$

Układy sprężyn





$r_1 = 20$ mm $\quad r_3 = 18$ mm $\quad G_{st} = 80 \times 10^9$ N/m^2

$r_2 = 25$ mm $\quad r_4 = 30$ mm $\quad G_{al} = 40 \times 10^9$ N/m^2

$$k_{eq} = \frac{1}{\sum_{i=1}^{n} \frac{1}{k_i}}$$

$$k_{t_{eq}} = \frac{1}{\frac{1}{k_{t_{AB_{al}}} + k_{t_{AB_{st}}}} + \frac{1}{k_{t_{BC}}}}$$

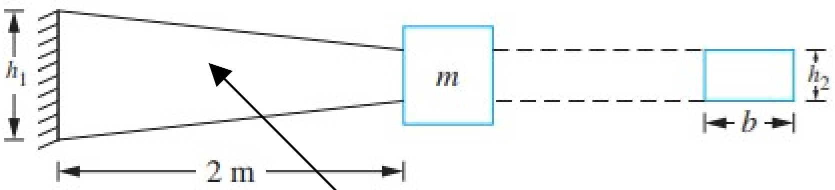
$k_t = JG/L$

$$k_{t_{AB_{al}}} = \frac{\frac{\pi}{32}(0.04\ \mathrm{m})^4\left(40 \times 10^9\ \frac{\mathrm{N}}{\mathrm{m}^2}\right)}{0.3\,\mathrm{m}} = 3.35 \times 10^4\ \frac{\mathrm{N \cdot m}}{\mathrm{rad}}$$

$$k_{t_{AB_{st}}} = \frac{\frac{\pi}{32}[(0.05\ \mathrm{m})^4 - (0.04\ \mathrm{m})^4]\left(80 \times 10^9 \frac{\mathrm{N}}{\mathrm{m}^2}\right)}{0.3\ \mathrm{m}} = 9.66 \times 10^4\ \frac{\mathrm{N \cdot m}}{\mathrm{rad}}$$

$$k_{t_{BC}} = = \frac{\frac{\pi}{32}[(0.06\,\mathrm{m})^4 - (0.036\,\mathrm{m})^4]\left(80 \times 10^9 \frac{\mathrm{N}}{\mathrm{m}^2}\right)}{0.2\,\mathrm{m}} = 4.43 \times 10^5\ \frac{\mathrm{N \cdot m}}{\mathrm{rad}}$$

$$k_{t,eq} = 1.01 \times 10^5\,\mathrm{N \cdot m/rad}$$



$h_2 = 20$ mm
$h_1 = 25$ mm
$b = 13$ mm
$E = 210 \times 10^9\ \text{N/m}^2$

$$\Delta = \int_0^L \frac{dz}{AE}$$

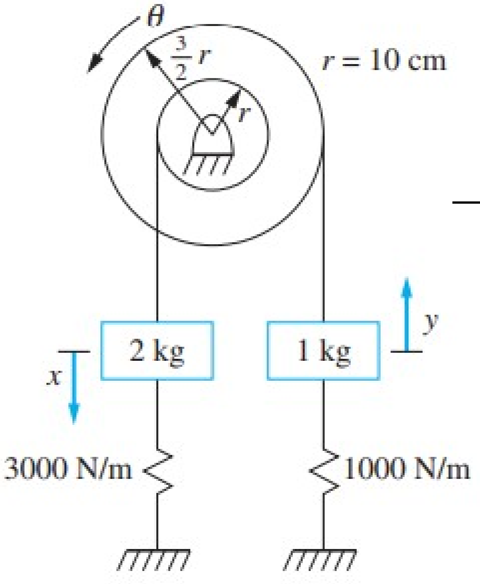
$$A = \left(h_1 - \frac{h_1 - h_2}{L} z\right) b.$$

$$\Delta = \frac{1}{bE}\int_0^L \frac{dz}{h_1 - \frac{h_1 - h_2}{L} z} = \frac{1}{bE}\left(\frac{-L}{h_1 - h_2}\right)\ln\left(h_1 - \frac{h_1 - h_2}{L} z\right)\Bigg|_0^L = \frac{L}{bE(h_1 - h_2)}\ln\left(\frac{h_1}{h_2}\right)$$

$$= \frac{2\,\text{m}}{(0.013\ \text{m})\ (210 \times 10^9\,\text{N/m}^2)(0.025\ \text{m} - 0.02\ \text{m})} \ln\frac{0.025\ \text{m}}{0.02\ \text{m}}$$

$$= 3.27 \times 10^{-8}\text{m/N}$$

$$k_{eq} = \frac{1}{\Delta} = \frac{1}{3.27 \times 10^{-8}\,\text{m/N}} = 3.06 \times 10^7\ \text{N/m}$$



$$\dot{x} = r\dot{\theta}$$

$$\dot{y} = \frac{3}{2}r\dot{\theta}$$

$$; y(0) = x(0) = 0$$

$$y = \frac{3}{2}x$$

Energia potencjalna zakumulowana w sprężynie

$$V = \sum_{i=2}^{n}\left[\frac{1}{2}k_i(\gamma_i x)^2\right]$$

$$= \frac{1}{2}\left(\sum_{i=1}^{n} k_i \alpha_i^2\right)x^2$$

$$= \frac{1}{2}k_{eq}x^2$$

$$V = \frac{1}{2}(3000\ \mathrm{N/m})x^2 + \frac{1}{2}(1000\ \mathrm{N/m})\left(\frac{3}{2}x\right)^2$$

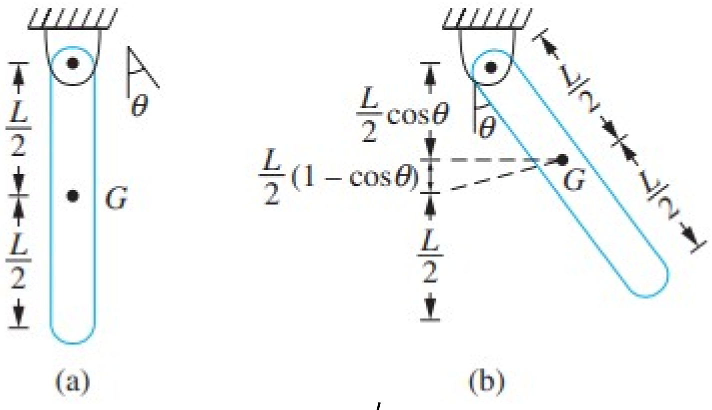
$$= \frac{1}{2}(5250\mathrm{N/m})x^2$$

grawitacja

$V = mgh$ Energia potencjalna

Energia potencjalna liczona względem O

$$V = -mg\frac{L}{2}\cos\theta$$

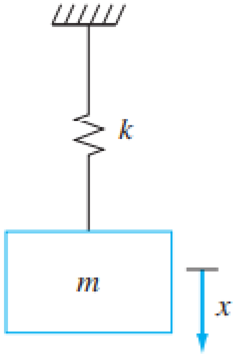


Energia potencjalna liczona względem G

$$h = \frac{L}{2} + \frac{L}{2}(1 - \cos\theta)$$

$$V = -mg\frac{L}{2}\cos\theta$$

$$V = mg\frac{L}{2}(2 - \cos\theta)$$



Statyczne ugięcie

$$\Delta = \frac{mg}{k}.$$

Energia potencjalna sprężyny

$$V = \frac{1}{2}k(x + \Delta)^2$$

Energia grawitacyjna

$$Vg = -\,mgx$$

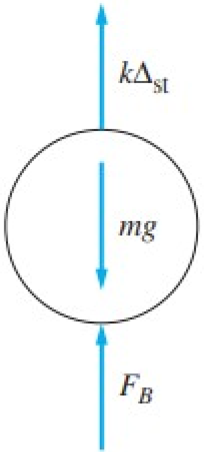
Całkowita energia

$$\begin{aligned} V &= \frac{1}{2}k(x + \Delta)^2 - mgx \\ &= \frac{1}{2}k\left(x + \frac{mg}{k}\right)^2 - mgx \\ &= \frac{1}{2}\left(kx^2 - 2mgx + \frac{m^2g^2}{k}\right) - mgx \\ &= \frac{1}{2}kx^2 + V_0 \end{aligned}$$

Energia sprężyny w stanie równowagi

$$V_0 = \frac{m^2g^2}{2k}$$

Pływalność w płynie



Kłania się Archimedes

$$k\Delta_{st} + F_B - mg = 0$$

$$F_B = \frac{4}{3}\rho g \pi r^3 = \frac{4}{3}(1200\ \mathrm{kg/m^3})\pi(9.81\ \mathrm{m/s^2})(0.1\ \mathrm{m})^3 = 49.3\ \mathrm{N}$$

$$\Delta_{st} = \frac{mg - F_B}{k} = \frac{(2.5\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) - 49.3\ \mathrm{N}}{1000\ \mathrm{N/m}} = -0.0185\ \mathrm{m}$$

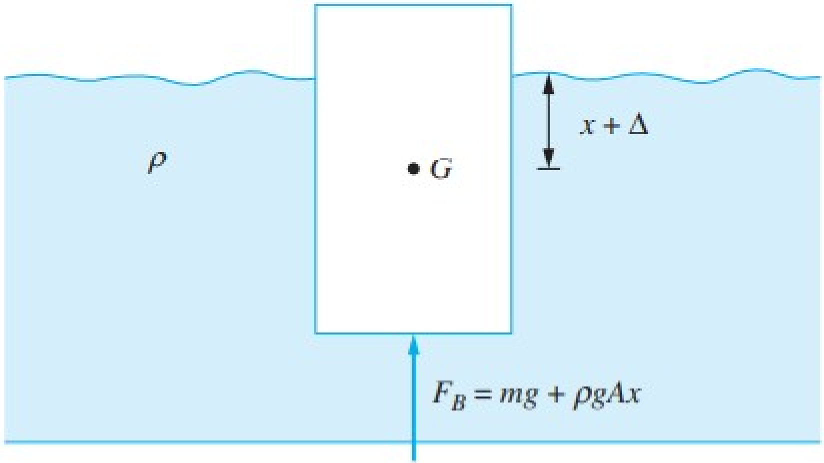
Pływalność w płynie

Kłania się Archimedes

x – przemieszczenie pionowe środka ciężkości
Delta odległość SC od powierzchni

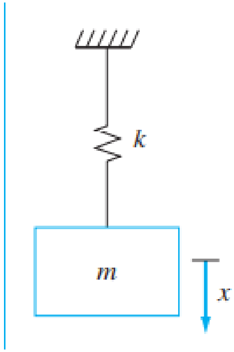
$$F_B = mg + \rho g A x$$

Praca sił wyporności (przy drganiu pionowym klocka) między x1 i x2



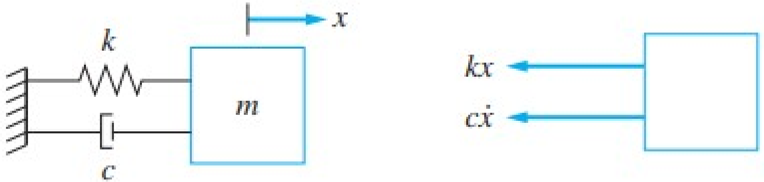
$$U_{1\to 2} = \frac{1}{2}\rho g A x_1^2 - \frac{1}{2}\rho g A x_2^2$$

Układ ten może być zastąpiony układem o SDOF



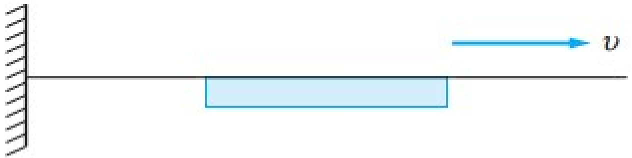
Tłumienie

Siła tłumiąca $F = cv$

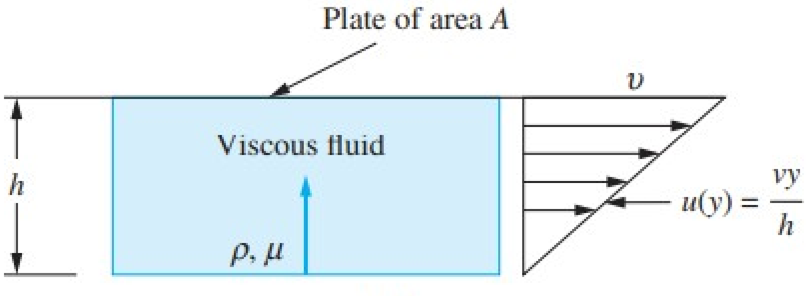


(a) (b)

Model tłumika



(a)



(b)

Prędkość płynu

$$u(y) = v\frac{y}{h}$$

Naprężene styczne

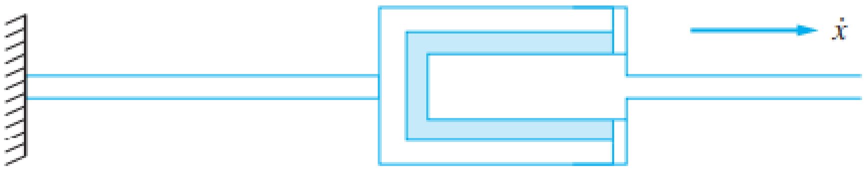
$$\tau = \mu\frac{du}{dy} = \mu\frac{v}{h}$$

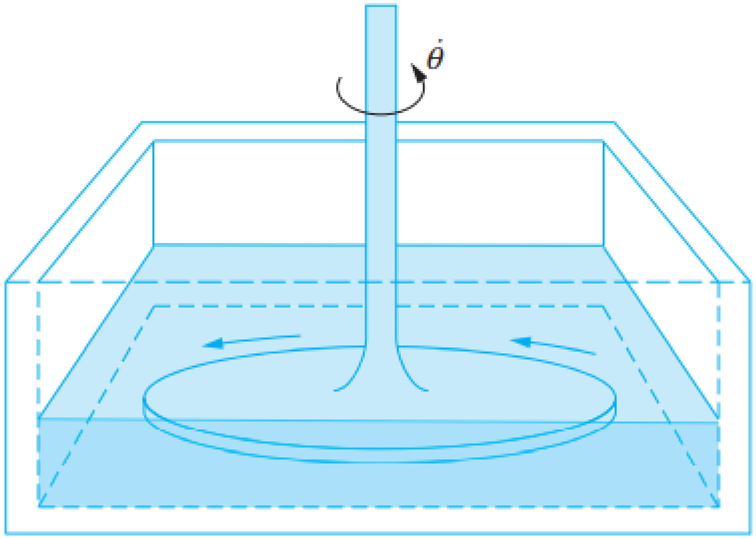
Siła

$$F = \tau A = \frac{\mu A}{h}v$$

$$c = \frac{\mu A}{h}$$

Tłumienie

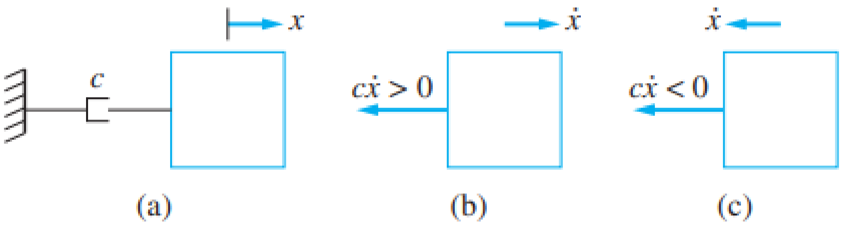




F=k xdot

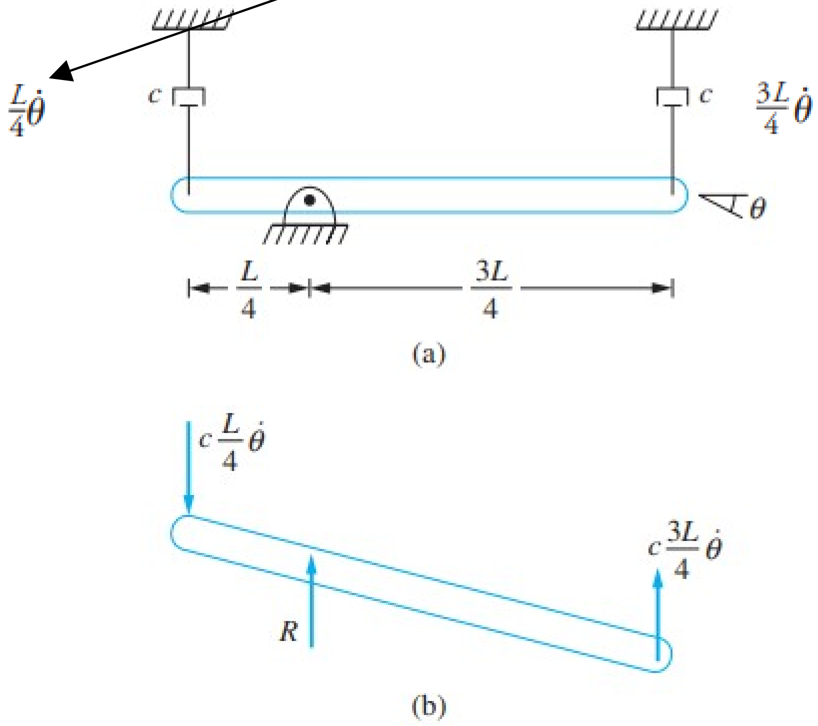
$$M = c_t \dot{\theta}$$

Tłumienie jest zawsze zgodne z zwrotem prędkosci



Równowaga układu z tłumieniem

Prędkości punktów A i B



Energia rozpraszana w sprężynie (Energy dissipated by viscous damping)

Z zasady zachowania energii mamy, że
Praca układu przy przejściu z stanu 1 do 2

$$U_{1\to 2_{NC}} = T_2 + V_2 - (T_1 + V_1)$$

Jeśli w układzie nie ma def. Sprężystych (odwracalnych)

$$U_{1\to 2} = -\int_0^x c\dot{x}\,dx$$

Dla układu wielu elementów tłumiących mamy:

$$U_{1\to 2} = -\sum_{i=1}^{n}\int_0^{x_i} c_i\dot{x}_i\,dx_i$$

$$x_i = \gamma_i x.$$

$$U_{1\to 2} = -\sum_{i=1}^{n}\int_0^{x} c_i(\gamma_i\dot{x})\,d(\gamma_i x)$$

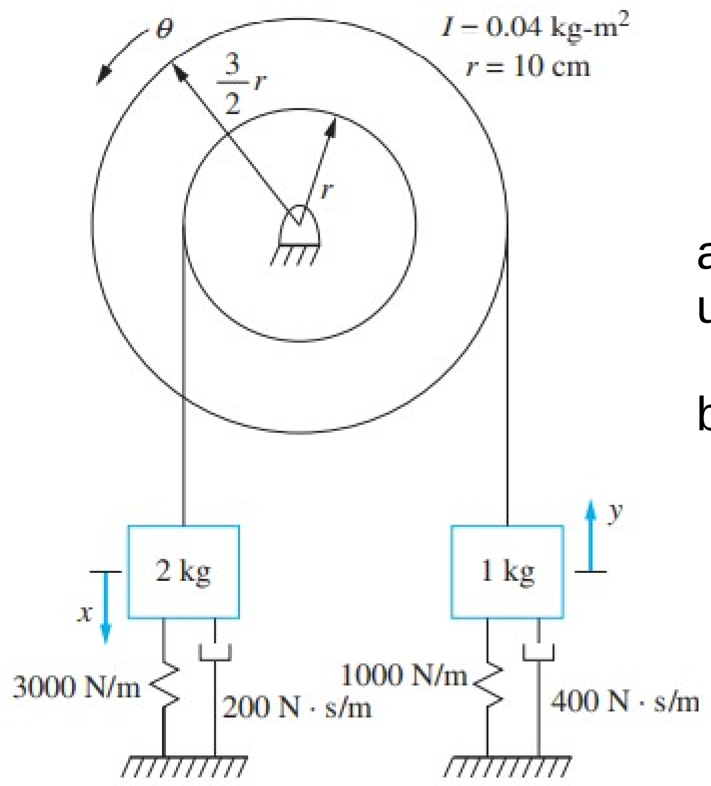
$$= -\sum_{i=1}^{n}\int_0^{x} c_i(\gamma_i^2\dot{x})\,dx$$

$$U_{1\to 2} = -\int_0^x\left(\sum_{i=1}^{n} c_i\gamma^2{}_i\right)\dot{x}dx$$

$$= -\int_0^x c_{\text{eq}}\,\dot{x}dx$$

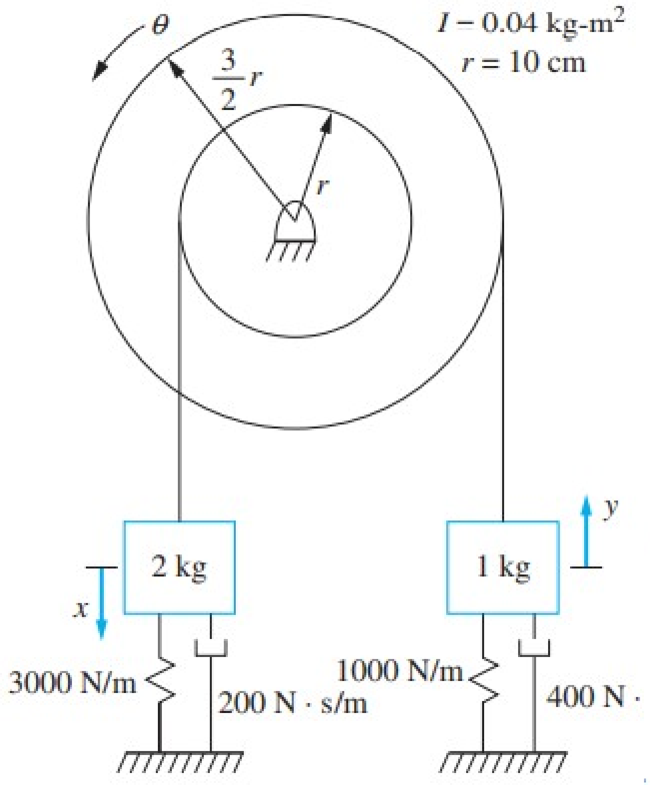
$$U_{1\to 2} = -\int_0^x c_{t,\text{eq}}\,\dot{\theta}\,d\theta$$

Dla tłumienia skrętnego



a) Określić ekwialentne tłumienie jeśli przemieszenie x jest uznane za współrzędną uogólnioną

b) Określić ekw. tłumienie jeśli obrót uznany za współrzędną uogólnioną



a) $y = \frac{3}{2}x.$

$$U_{1\to 2} = -\int_0^x (200\ \mathrm{N\cdot s/m})\ \dot{x}\ dx - \int_0^x (400\,\mathrm{N\cdot s/m}) \left(\frac{3}{2}\dot{x}\right) d\left(\frac{3}{2}x\right)$$

$$= -\int_0^x (1100\ \mathrm{N\cdot s/m})\ \dot{x}\ dx$$

$$c_{eq} = 1100\ \mathrm{N\cdot s/n}$$

b)

$$x = r\theta \qquad y = \frac{3}{2} r\theta \qquad r = 0.1\ \mathrm{m.}$$

$$U_{1\to 2} = -\int_0^\theta (200\ \mathrm{N\cdot s/m})[(0.1\mathrm{m})\dot{\theta}]\,d[(0.1\ \mathrm{m})\theta] - \int_0^\theta (400\,\mathrm{N\cdot s/m})\left[\frac{3}{2}(0.1\ \mathrm{m})\dot{\theta}\right]$$

$$\times\ d\left[\frac{3}{2}(0.1\,\mathrm{m})\theta\right] = -\int_0^\theta \left(11\frac{\mathrm{N\cdot m\cdot s}}{\mathrm{rad}}\right)\dot{\theta}\,d\theta$$

$$c_{t,eq} = 11\ \mathrm{N\cdot m\cdot s/rad}$$

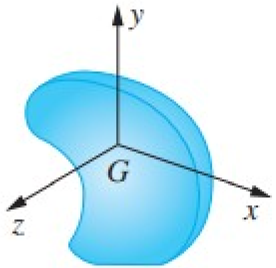
Siły bezwładności

Masowy moment bezwładności

$$\bar{I} = \int_m [(x - \bar{x})^2 + (y - \bar{y})]^2 \, dm$$

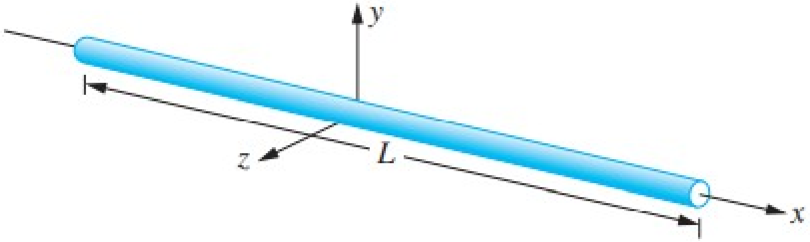
gdzie $(\bar{x}, \bar{y})$ to wsp. środka ciężkości

General shape



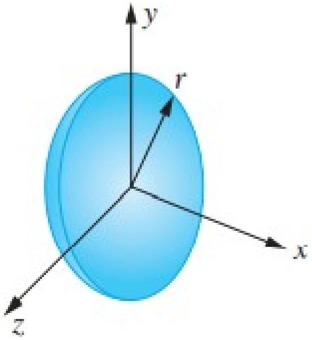
$$\bar{I}_x = \int (y^2 + z^2)\,dm$$
$$\bar{I}_y = \int (x^2 + z^2)\,dm$$
$$\bar{I}_z = \int (x^2 + y^2)\,dm$$

Slender rod



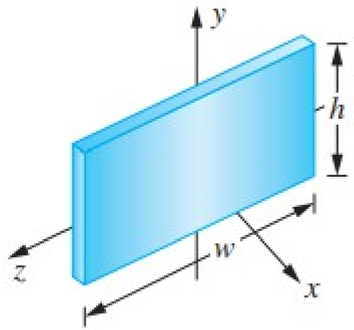
$$\bar{I}_x \approx 0$$
$$\bar{I}_y = \frac{1}{12}mL^2$$
$$\bar{I}_z = \frac{1}{12}mL^2$$

Thin disk



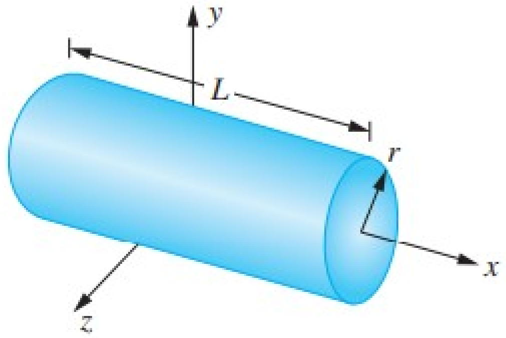
$$\bar{I}_x = \frac{1}{2}mr^2$$
$$\bar{I}_y = \frac{1}{4}mr^2$$
$$\bar{I}_z = \frac{1}{4}mr^2$$

Thin plate



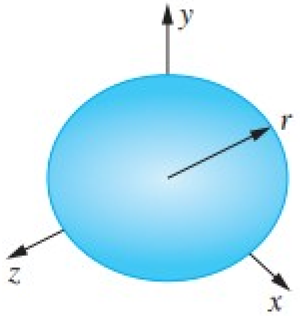
$$\bar{I}_x = \frac{1}{12}m(w^2 + h^2)$$
$$\bar{I}_y = \frac{1}{12}mw^2$$
$$\bar{I}_z = \frac{1}{12}mh^2$$

Circular cylinder



$$\bar{I}_x = \frac{1}{12}mr^2$$
$$\bar{I}_y = \frac{1}{12}m(3r^2 + L^2)$$
$$\bar{I}_z = \frac{1}{12}m(3r^2 + L^2)$$

Sphere



$$\bar{I}_x = \frac{2}{5}mr^2$$
$$\bar{I}_y = \frac{2}{5}mr^2$$
$$\bar{I}_z = \frac{2}{5}mr^2$$

Energia kinetyczna cząstki

$$\frac{1}{2}mv^2$$

Energia kinetyczna ciała sztywnego
w ruchu płaskim

$$\frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2.$$

Ruch postępowy

Ruch obrotowy

Całkowita energia kinetyczna

$$T = \sum_{i=1}^{n}\left(\frac{1}{2}m_i v_i^2 + \frac{1}{2}\bar{I}_i\omega_i^2\right)$$

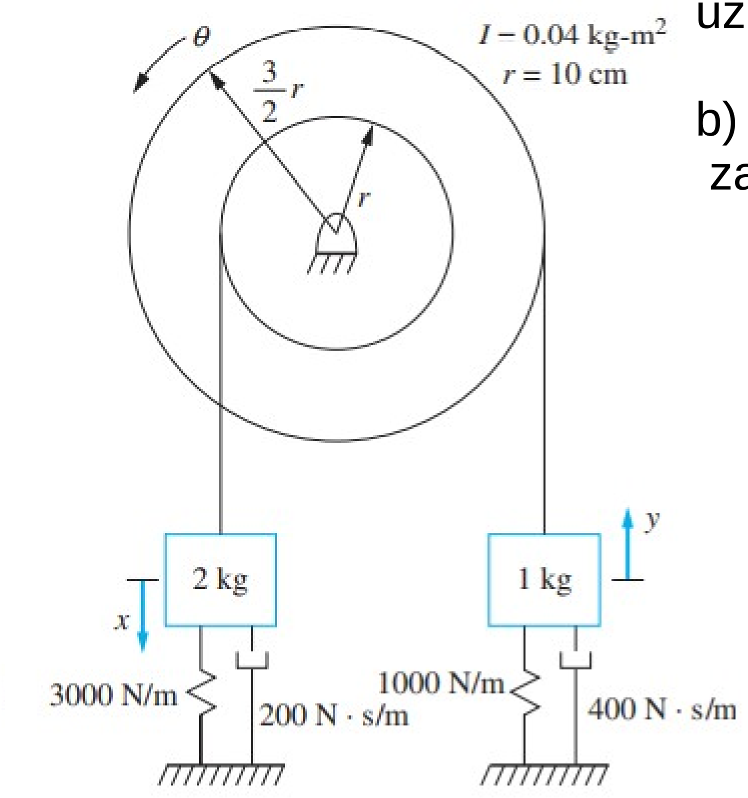
$$\sum_{i=1}^{n}\left[\frac{1}{2}m_i(\beta_i\dot{x})^2 + \frac{1}{2}\bar{I}_i(\nu_i\dot{x})^2\right]$$

$$= \frac{1}{2}\left[\sum_{i=1}^{n}(m_i\beta_i^2 + \bar{I}_i\nu_i^2)\right]\dot{x}^2$$

$$= \frac{1}{2}m_{\mathrm{eq}}\dot{x}^2$$

Tylko dla ruchu obrotowego

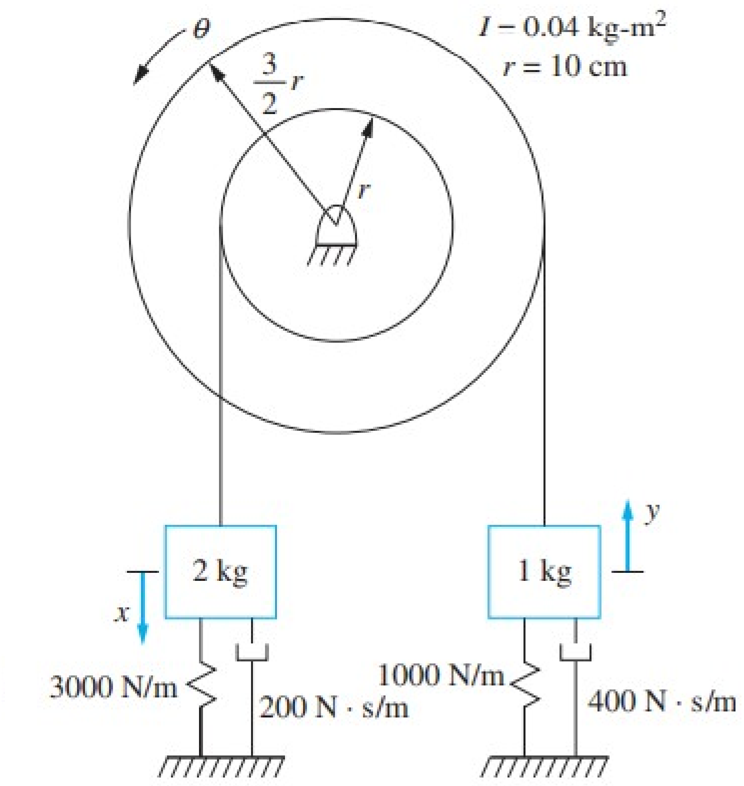
$$T = \frac{1}{2}I_{\mathrm{eq}}\dot{\theta}^2$$



a) Określić ekwialentną masę jeśli przemieszenie x jest uznane za współrzędną uogólnioną

b) Określić ekw. masę jeśli obrót uznany za współrzędną uogólnioną

a) $y = \frac{3}{2}x \text{ and } \theta = \frac{x}{r} = \frac{x}{0.1\ \text{m}} = 10\,x.$



$$T = \frac{1}{2}(2\ \text{kg})\dot{x}^2 + \frac{1}{2}(1\ \text{kg})\dot{y}^2 + \frac{1}{2}(0.0\,4\ \text{kg}\cdot\text{m}^2)\dot{\theta}^2$$

$$= \frac{1}{2}(2\ \text{kg})\dot{x}^2 + \frac{1}{2}(1\ \text{kg})\left(\frac{3}{2}\dot{x}\right)^2 + \frac{1}{2}(0.04\ \text{kg}\cdot\text{m}^2)(10\dot{x}\,\text{m}^{-1})^2$$

$$= \frac{1}{2}(8.25\ \text{kg})\dot{x}^2$$

b) $y = \frac{3}{2}r\theta = \frac{3}{2}(0.1\ \text{m})\theta$

$$T = \frac{1}{2}(2\ \text{kg})\dot{x}^2 + \frac{1}{2}(1\ \text{kg})\dot{y}^2 + \frac{1}{2}(0.04\ \text{kg}\cdot\text{m}^2)\dot{\theta}^2$$

$$= \frac{1}{2}(2\ \text{kg})[(0.1\text{m})\dot{\theta}]^2 + \frac{1}{2}(1\text{kg})\left[\frac{3}{2}(0.1\ \text{m})\dot{\theta}\right]^2 + \frac{1}{2}(0.04\ \text{kg}\cdot\text{m}^2)\dot{\theta}^2$$

$$= \frac{1}{2}(0.0825\ \text{kg}\cdot\text{m}^2)\dot{\theta}^2$$